The Optimal Inflation Target in an Economy with Limited Enforcement

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* Views expressed are those of the author and do not necessarily reflect official positions of the FOMC or the Federal Reserve System.

Comfort zones

- Typical FOMC member statements: 1 to 2 percent inflation would be ideal.
  - "Small positive inflation is the best inflation target."
- Associated with short-term nominal interest rates near 5 percent.
- Economic theory: nominal interest rates should be zero.
- Why the difference?

Theory and reality: Why the difference?

- Japan.
- Summers (1991): Zero bound on nominal interest rates interferes with stabilization policy.
    - Adam and Billi (2006).
- Smith (2002). Deflation causes disintermediation, harming the operation of credit markets.
- We study a version of the latter:
    - A small amount of inflation "deepens financial markets" in a way we make precise.

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- Two assets: currency and consumption loans.

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- Inflation will make default less attractive, loosening participation constraints and strengthening the credit market.

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- If the planner puts a relative weight greater than zero but less than the population weight on credit households, the optimum inflation rate is positive.
- Conclude: Independent central banks will set low positive inflation targets in economies that possess highly developed financial markets.

Recent related literature

- Aiyagari and Williamson (2000 *JET*).
  - Related environment, random endowments, outside option is Bewley, emphasis on financial intermediation, positive inflation deters default, computational.
- Berentsen, Camera and Waller (2007 *JET*).
  - Related environment, Lagos-Wright, emphasis on financial intermediation, positive inflation deters default.
- Ragot (2007). OLG framework.
- Andolfatto (2007).
- Antinolfi, Azariadis, and Bullard (2007).
  - Money is the only asset, emphasis on dynamics and equilibrium selection.
- Sanchez, Williamson, and Wright (2007). Blackboard version.

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- λ "Kehoe-Levine" or *credit* type agents.
- $1-\lambda$ "Townsend-Bewley" or *cash* type agents.
- All types have identical preferences

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \tag{1}$$

with discount factor $0 < \beta < 1$, $u(c)$ standard, and $i = 0, 1, 2, 3$.

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- Endowments, interpreted as income shares, are periodic:

$$\left(\omega_t^0, \omega_t^1\right) = \left(\omega_t^2, \omega_t^3\right) = \begin{cases} (1+\alpha, 1-\alpha) & \text{if } t = 0, 2, \ldots \\ (1-\alpha, 1+\alpha) & \text{if } t = 1, 3, \ldots \end{cases} \tag{2}$$

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- Aggregate income is constant.

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- Future inflation rates will impact the payoff to default.

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- ... and directly selects consumption vectors for credit agents who may either accept their allocations or behave like cash agents in perpetuity.
- The inflation target in this economy is similar to an optimal tax subject to an incentive constraint as understood by Mirrlees (1971).

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  - a reduction in the default payoff which brings about higher debt limits.

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- Because inflation is a distortionary tax, we define a *modified planning problem*.

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- Next, given π, high income cash agents choose a periodic consumption vector $(x_H, x_L) \geq 0$ to maximize stationary discounted utility

$$\frac{1}{1-\beta^2}\left[u\left(x_H\right)+\beta u\left(x_L\right)\right] \tag{3}$$

subject to

$$x_H \leq 1+\alpha, \tag{4}$$

$$x_H+\pi x_L=1+\alpha+\pi\left(1-\alpha\right), \tag{5}$$

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- (1) nonnegative currency; (2) money balances are used up to smooth consumption in low income dates; (3) agents can renounce money and consume endowments.

More on the modified planning problem

- Let $x_H(\pi)$ and $x_L(\pi)$ solve the previous problem. Given π, the planner now chooses consumption values $(c_H, c_L) \geq 0$ for credit households to maximize the equal-treatment welfare function

$$\frac{1}{1-\beta^2}\left[u(c_H) + u(c_L)\right] \tag{7}$$

subject to the resource constraint

$$\lambda(c_H + c_L) + (1-\lambda)\left[x_H(\pi) + x_L(\pi)\right] = 2, \tag{8}$$

and the participation constraint

$$u(c_H) + \beta u(c_L) \geq u\left[x_H(\pi)\right] + \beta u\left[x_L(\pi)\right]. \tag{9}$$

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- Equal treatment of high income and low income households means that the discounted utilities are weighted equally. This gives the welfare function above.

More on the modified planning problem

- If $c_H(\pi)$ and $c_L(\pi)$ solve the previous problem for a given $\pi > 0$, the planner selects the stationary inflation factor π to maximize the social welfare function

$$\mathcal{W}(\pi,\delta) = \delta\{u[c_H(\pi)] + u[c_L(\pi)]\} + (1-\delta)\{u[x_H(\pi)] + u[x_L(\pi)]\}. \quad (10)$$

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- A strictly utilitarian SWF would have equal weights for all, that is, $\delta = \lambda$.

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- This gives a *first best* solution with $\pi = \beta$, $R^N = 1$, and smooth consumption for both groups of agents.

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- Suppose $\delta = 1$, no welfare weight on the cash community. Then the planner chooses maximal seigniorage $\tilde{\pi}$ and smooths the consumption of the credit community completely.
- Suppose $\delta = 0$, no welfare weight on the credit community. Then the planner will push the inflation factor as close to zero as possible.

More on second best

- The second best trades off these two extreme cases.

Theorem

The second best optimum allocation under a utilitarian social welfare function satisfies $(c_H, c_L, x_H, x_L) = (c^{**}, c^{**}, x_H(\pi^{**}), x_L(\pi^{**}))$. *It is supported by a "modified Friedman rule" for some inflation factor* $\pi^{**} \in (\beta, 1)$, *and a nominal yield such that* $R^N = \pi^{**}/\beta > 1$.

Proof.

See the appendix. □

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    - Perhaps more controversial?
    - "The market for unsecured credit can be made to work perfectly."

Lemma 2

Lemma

Define $\mathcal{W}_{\pi}(\pi, \delta) = \partial \mathcal{W} / \partial \pi$. *Then (a)* $\mathcal{W}_{\pi}(\pi, \delta) < 0$ $\forall (\pi, \delta) \in [\bar{\pi}, \tilde{\pi}] \times [0, \lambda]$, *and (b)* $lim_{\pi \to 1} \mathcal{W}_{\pi}(\pi, \delta) = +\infty$ *when* π *converges from above.*

- Part (*a*) intuition. To raise π above $\bar{\pi}$ does not improve the ability of the planner to smooth the consumption of the credit community any further. Doing so merely transfers income from the cash community. This transfer will reduce social welfare except when $\delta > \lambda$.

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- Part (*b*) intuition. A small increase in the inflation tax creates a credit market where one would otherwise not exist.

Lemma 3

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$\mathcal{W}(\pi, \delta)$ is not defined for $\pi < 1$. It is decreasing in π for $\pi \in (\tilde{\pi}, 1/\bar{R})$ and constant for $\pi \geq 1/\bar{R}$.

- Deflation violates the participation constraint for high income credit households.

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- Deflation violates the participation constraint for high income credit households.
- The outcome of any deflation is that money has a higher payoff than credit.

Theorem 4

Theorem

Suppose assumptions **A1**, **A2**, *and* **A3** *hold, and* $0 < \delta \leq \lambda$. *Then the optimum inflation factor is* $\pi^{\star}(\delta) > 1$ *and the associated nominal interest yield,* $R^N \in (\pi^{\star}(\delta), \pi^{\star}(\delta)/\beta)$, *is even higher.*

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- Sounds like “comfort zones” articulated by central bankers.