



PANEL DISCUSSION: THE ROLE OF POTENTIAL OUTPUT IN POLICYMAKING

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Views expressed are those of the author and do not necessarily reflect official positions of the FOMC or the Federal Reserve System.



MY DISCUSSION

- Describe ideas about “proper” detrending.
- Core idea requires explicit theory of both growth and fluctuations.
- Ambition: The data should then be detrended by the theoretical growth path.
 - Question: How to get the growth path to look like the data?
 - Answer: Simple growth model with occasional trend breaks and learning.
- Applications in RBC and NK models.
 - J. Bullard and J. Duffy. “Learning and Structural Change in Macroeconomic Data.”
 - J. Bullard and S. Eusepi. “Did the Great Inflation Occur Despite Policymaker Commitment to a Taylor Rule?”
- Policy: How Taylor rules can lead you astray.



MAIN IDEAS

- Equilibrium business cycle research: A wide class of models including RBC, NK.
- All based on the concept of a balanced growth path.
- Data as summarized by Perron (1989) and Hansen(2001) suggest breaks in trends.
- Nonstationary aspects of the data are difficult to reconcile with available models.



CURRENT PRACTICE

- Trend-cycle decomposition done mostly with atheoretic, statistical filters. See King and Rebelo (1999).
- The discipline implied by the balanced growth assumption is dropped.
- This is a mistake, but one that dominates the literature.



SOME WELL-KNOWN CRITICISMS

- Filters do not remove the same trend that the balanced growth path requires.
- Current practice does not respect the cointegration of the variables, the multivariate trend, that the model implies.
- Filtered trends imply changes in growth rates, and agents would want to react to these changes.
- The "business cycle facts" are not independent of the statistical filter employed.
- Estimation, e.g., Smets-Wouters does not address this issue.



HOW TO IMPROVE ON THIS?

- The criticisms are correct in principle.
- They are quantitatively important.
- These issues cannot be resolved by alternative statistical filters, because those filters are atheoretic.
- Instead, use theory to tell us what the growth path should look like.



CORE IDEAS

- "Model-consistent detrending."
- The trends removed from the data are exactly the same as the ones implied by the model.
- Allow agents to react to (rare) changes in trend growth rates.
- Respect the cointegration of the variables that the balanced growth path implies.
- The methodology has wide applicability.



FEATURES OF THE ENVIRONMENT

- Simple equilibrium business cycle model with exogenous, stochastic growth.
- Replace rational expectations with learning via Evans and Honkapohja (2001).
- Verify expectational stability.
- Calibrate, allowing occasional trend breaks, inspired by Perron (1989).



MAIN FINDINGS

- A more satisfactory method of detrending.
- A large fraction of the observed variance of output relative to trend can be attributed to structural change.
- In the NK world, learning about a productivity slowdown can send inflation up by 300 b.p.



PREFERENCES

- As in Cooley and Prescott (1994), a representative household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \eta^t \left[\ln C_t + \theta \ln (1 - \hat{\ell}_t) \right] \quad (1)$$

- subject to

$$C_t + I_t \leq Y_t, \quad (2)$$

$$I_t = K_{t+1} - (1 - \delta) K_t, \quad (3)$$

- and ...



TECHNOLOGY

- The production technology is

$$Y_t = \hat{s}_t K_t^\alpha \left(X_t N_t \hat{\ell}_t \right)^{1-\alpha}, \quad (4)$$

$$X_t = \gamma X_{t-1}, \quad X_0 = 1, \quad \gamma > 0. \quad (5)$$

$$N_t = \eta N_{t-1}, \quad N_0 = 1, \quad \eta > 0. \quad (6)$$

$$\hat{s}_t = \hat{s}_{t-1}^\rho \epsilon_t, \quad \hat{s}_0 = 1, \quad (7)$$



BALANCED GROWTH

- Aggregate output, consumption, investment, and capital will grow at gross rate $\gamma\eta$ along the balanced growth path.
- Define $\hat{k}_t = \frac{K_t}{X_t N_t}$, $\hat{y}_t = \frac{Y_t}{X_t N_t}$, $\hat{c}_t = \frac{C_t}{X_t N_t}$, and rewrite the first order conditions and constraints of the problem.
- The new, stationary system has a steady state $(\hat{c}_t, \hat{y}_t, \hat{k}_t, \hat{\ell}_t) = (\bar{c}, \bar{y}, \bar{k}, \bar{\ell}) \forall t$.
- The steady state values depend upon the gross growth rates γ and η .



KEY RATIOS

- Capital-output ratio along a balanced growth path

$$\frac{\bar{k}}{\bar{y}} = \frac{\alpha\beta}{\gamma - \beta(1 - \delta)}, \quad (8)$$

- Consumption-output ratio along a balanced growth path

$$\frac{\bar{c}}{\bar{y}} = \frac{\gamma - \beta(1 - \delta) - \alpha\beta(\gamma\eta - 1 + \delta)}{\gamma - \beta(1 - \delta)}. \quad (9)$$



LINEAR APPROXIMATION

- Need a linear system to apply Evans and Honkapohja (2001).
- Use logarithmic deviations from steady state.
- Define

$$\tilde{c}_t = \ln \left(\frac{\hat{c}_t}{\bar{c}} \right), \quad \tilde{k}_t = \ln \left(\frac{\hat{k}_t}{\bar{k}} \right), \quad \tilde{\ell}_t = \ln \left(\frac{\hat{\ell}_t}{\bar{\ell}} \right), \quad (10)$$

$$\tilde{y}_t = \ln \left(\frac{\hat{y}_t}{\bar{y}} \right), \text{ and } \tilde{s}_t = \ln \left(\frac{\hat{s}_t}{\bar{s}} \right). \quad (11)$$

- Rewrite system in terms of tilde variables.



MORE ON LINEAR APPROXIMATION

- The linearized system is still not satisfactory, because the log deviations involve $(\bar{c}, \bar{y}, \bar{k}, \bar{\ell})$.
- Want the agents to learn the vector $(\bar{c}, \bar{y}, \bar{k}, \bar{\ell})$ when a change in growth occurs.
- Define $c_t = \ln \hat{c}_t$, $k_t = \ln \hat{k}_t$, $y_t = \ln \hat{y}_t$, $\ell_t = \ln \hat{\ell}_t$, and $s_t = \ln \hat{s}_t$.
- Also define $c = \ln \bar{c}$, $k = \ln \bar{k}$, $y = \ln \bar{y}$, $\ell = \ln \bar{\ell}$, and $s = \ln \bar{s} = 0$.
- Rewrite the system in terms of these redefined variables; reduce system to three equations.



THE SYSTEM UNDER RATIONAL EXPECTATIONS

- The system:

$$c_t = \mathcal{B}_{10} + \mathcal{B}_{11}E_t c_{t+1} + \mathcal{B}_{12}E_t k_{t+1} + \mathcal{B}_{13}E_t s_{t+1} \quad (12)$$

$$k_t = \mathcal{D}_{20} + \mathcal{D}_{21}c_{t-1} + \mathcal{D}_{22}k_{t-1} + \mathcal{D}_{23}s_{t-1} \quad (13)$$

$$s_t = \rho s_{t-1} + \vartheta_t \quad (14)$$

- The \mathcal{B}_{ij} and \mathcal{D}_{ij} are composites of fundamental parameters. Also, $\vartheta_t = \ln \epsilon_t$.



RECURSIVE LEARNING

- Study this system under a recursive learning assumption.
- Assume agents have no specific knowledge of the economy.
- Endow them with a perceived law of motion which looks a lot like a VAR.



MORE ON RECURSIVE LEARNING

- The system:

$$c_t = \mathcal{B}_{10} + \mathcal{B}_{11}E_t^*c_{t+1} + \mathcal{B}_{12}E_t^*k_{t+1} + \mathcal{B}_{13}E_t^*s_{t+1} + \Delta_t \quad (15)$$

$$k_t = \mathcal{D}_{20} + \mathcal{D}_{21}c_{t-1} + \mathcal{D}_{22}k_{t-1} + \mathcal{D}_{23}s_{t-1} \quad (16)$$

$$s_t = \rho s_{t-1} + \vartheta_t \quad (17)$$

- The shock Δ_t prevents perfect multicollinearity. It has standard deviation $1/1000th$ of ϵ_t .



THE PERCEIVED LAW OF MOTION

- The agents use

$$c_t = a_{10} + a_{11}c_{t-1} + a_{12}k_{t-1} + a_{13}s_{t-1}, \quad (18)$$

$$k_t = a_{20} + a_{21}c_{t-1} + a_{22}k_{t-1} + a_{23}s_{t-1}. \quad (19)$$

- This corresponds in form to the equilibrium law of motion for the economy.
- The agents are given equation (17). They could estimate ρ as well without materially changing the results.
- The presence of constant terms allows the agents to learn steady state values of variables.



THE MAPPING FROM PLM TO ALM

- Assume $t - 1$ dating of expectations.
- Take expectations based on the PLM.
- Substitute to obtain the actual law of motion (ALM)



MORE ON THE MAPPING FROM PLM TO ALM

- This implies

$$c_t = T_{10} + T_{11}c_{t-1} + T_{12}k_{t-1} + T_{13}s_{t-1} + \Delta_t \quad (20)$$

- where

$$T_{10} = \mathcal{B}_{10} + \mathcal{B}_{11} [a_{10} + a_{11}a_{10} + a_{12}a_{20}] + \mathcal{B}_{12} [a_{20} + a_{21}a_{10} + a_{22}a_{20}], \quad (21)$$

$$T_{11} = \mathcal{B}_{11} [a_{11}^2 + a_{12}a_{21}] + \mathcal{B}_{12} [a_{21}a_{11} + a_{22}a_{21}], \quad (22)$$

$$T_{12} = \mathcal{B}_{11} [a_{11}a_{12} + a_{12}a_{22}] + \mathcal{B}_{12} [a_{21}a_{12} + a_{22}^2], \quad (23)$$

$$T_{13} = \mathcal{B}_{11} [a_{11}a_{13} + a_{12}a_{23} + a_{13}\rho] + \mathcal{B}_{12} [a_{21}a_{13} + a_{22}a_{23} + a_{23}\rho] + \mathcal{B}_{13} [\rho^2]. \quad (24)$$



THE SYSTEM UNDER LEARNING

- Write

$$\begin{bmatrix} c_t \\ k_t \\ s_t \end{bmatrix} = \begin{bmatrix} T_{10} \\ \mathcal{D}_{20} \\ 0 \end{bmatrix} + \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ \mathcal{D}_{21} & \mathcal{D}_{22} & \mathcal{D}_{23} \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} c_{t-1} \\ k_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta_t \\ 0 \\ \vartheta_t \end{bmatrix}. \quad (25)$$



RATIONAL EXPECTATIONS

- A stationary MSV solution solves

$$T_{1i} = a_{1i}, \quad (26)$$

for $i = 0, 1, 2, 3$, with all eigenvalues of the matrix

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ \mathcal{D}_{21} & \mathcal{D}_{22} & \mathcal{D}_{23} \\ 0 & 0 & \rho \end{bmatrix} \quad (27)$$

inside the unit circle.

- There is only one such solution for this model.



EXPECTATIONAL STABILITY

- Expectational stability is determined by the following matrix differential equation

$$\frac{d}{d\tau}(a) = T(a) - a, \quad (28)$$

where $T = (T_{10}, T_{11}, T_{12}, T_{13})$ and $a = a_{ij}$ with $i = 1, 2$ and $j = 1, 2, 3, 4$.

- A particular MSV solution is E -stable if the MSV fixed point of the differential equation (28) is locally asymptotically stable at that point.
- Calculated E -stability for this model and found that it holds at baseline parameter values (including the various values of γ and η used).
- A real time learning version can be implemented.



STABILITY UNDER CONSTANT GAIN LEARNING

- The system should be locally stable in the real time learning dynamics with gain of t^{-1} .
- With a constant gain, the system may depart the domain of attraction.
- But the constant gain also allows the agents to track the balanced growth path, should an underlying parameter change unexpectedly.
- The agents admit their model may be misspecified.
- How would the system respond to any small enough parameter change?



AN APPLICATION TO US DATA

- Model is too simple to match directly with data.
- But it is also a well-known benchmark model.
- So it is possible to assess how important the detrending issue is for determining the nature of the business cycle as well as for the performance of the model relative to the data.



DATA

- Quarterly U.S. data 1948Q1 to 2002Q1.
- Concern that the aggregates add up.
- Subtract real government purchases and farm business product from real GDP to get nonfarm private sector output.
- Using nonfarm private sector hours from the establishment survey.



MORE ON THE DATA

- Nonfarm private sector productivity created from these.
- Consumption defined as personal consumption expenditures for nondurable goods and services, plus net exports of services, less farm business product.
- Investment defined as gross private domestic investment plus personal consumption expenditures on consumer durables, plus net exports of goods.
- Series essentially add up despite chain-weighting. Allocated discrepancies using the consumption-output ratio for that year.



A STANDARD CALIBRATION

- Cooley and Prescott (1994).
- $\beta = .987, \theta = 1.78, \alpha = .4, \rho = .95, \sigma_\epsilon = .007$.
- Growth rates of productivity and labor input: allow those to change.
- Gain chosen informally at $g = .00025$; does not seem to impact results importantly.



BREAKS ALONG THE BALANCED GROWTH PATH

- Productivity slowdown: Hansen (2001), Perron (1989), Bai, Lumsdaine, and Stock (1998).
- Only allow trend breaks where clear econometric evidence is available?



HOW TO CHOOSE BREAK DATES

- One approach: conformity between *measured* productivity and labor input, in the model and in the data.
 - ① Choose break dates and growth rates.
 - ② Compare implied trends in measured productivity and labor input to data.
 - ③ If discrepancies exist, return to 1, otherwise terminate at a fixed point.
- Use a search process (genetic algorithm) to implement this process.



OPTIMAL TREND BREAKS

	$N(t)$	$X(t)$
Initial annual growth rate, percent	1.20	2.47
Break date	1961, Q2	1973, Q3
Mid-sample annual growth rate, percent	1.91	1.21
Break date	—	1993, Q3
Ending annual growth rate, percent	1.91	1.86

TABLE: Optimal break dates and growth rates for fundamental factors driving growth in the model, based on a search of possible dates and growth rates. These choices produce measured productivity and hours series that conform best to the US data.



DEFINING A TREND

- A trend is the economy's path if only low frequency shocks occur.
- Turn the noise on the business cycle shock down, multiplying σ_e by 1/1000.
- What happens in the economy where the only meaningful shocks are those to productivity growth and hours growth?
- Normalization: initially on a balanced growth path.



ARTIFICIAL ECONOMIES

- Confirm that estimated coefficients are initially close to RE values.
- Collect an additional 217 quarters of data, with trends breaking as described above.
- Detrend the data using the same (multivariate) trend that is used for the actual data.
- Collect statistics over a large number of simulations.



BUSINESS CYCLE STATISTICS

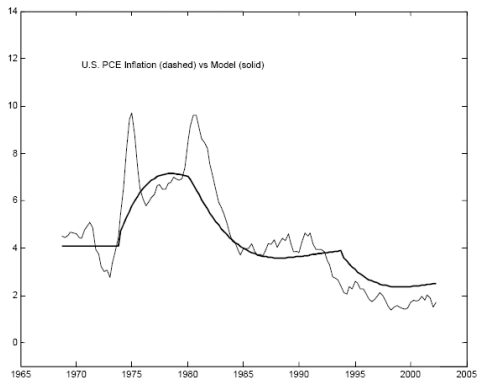
TABLE 3. BUSINESS CYCLE STATISTICS

	Volatility		Relative Volatility		Contemporaneous Correlations	
	Data	Model	Data	Model	Data	Model
Output	3.25	3.50	1.00	1.00	1.00	1.00
Consumption	3.40	2.16	1.05	0.62	0.60	0.75
Investment	14.80	8.86	4.57	2.53	0.65	0.92
Hours	2.62	1.54	0.81	0.44	0.65	0.80
Productivity	2.52	2.44	0.77	0.70	0.61	0.92

TABLE: Business cycle statistics, model-consistent detrending.



NK APPLICATION



If the central bank unexpectedly lowers the inflation target to two percent in 1980, the inflation dynamics begin to approximate the data quite well.



SUMMARY

- Provides some microfoundations for current, atheoretical practices.
- Structural change accounts for a large fraction of observed variability of output.
- Learning provides the “glue” that holds the various balanced growth paths together.
- Adjustment following a trend change is relatively rapid.
- Learning about structural change could have large effects on policy.