An Illustrative Calculation of $r^*$ with Policy Implications

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Any opinions expressed here are my own and do not necessarily reflect those of the Federal Open Market Committee.
Introduction
Why worry about $r^\dagger$?

• In a Taylor-type rule, the natural real interest rate, $r^\dagger$, determines the intercept:
  \[ i = r^\dagger + \pi^e + \phi_\pi \pi^{GAP} + \phi_y y^{GAP}, \]
  where $\pi^e = \pi^* = 2$ percent, the FOMC’s inflation target.

• When the gaps are zero, a Taylor-type rule simply recommends setting the policy rate equal to the value of $r^\dagger$ plus the inflation target.

• But what is the value of $r^\dagger$?
Decomposing $r^*$

- $r^*$ is often referred to as “the natural real rate of interest.”
- One way to think of it is to divide it into three factors:
  \[ r^* = \lambda + \psi + \xi, \quad \text{where} \]
- $\lambda$: the labor productivity growth rate
- $\psi$: the labor force growth rate
- $\xi$: an investor desire for safe assets. A strong desire for safe assets would imply a relatively large negative value for $\xi$, whereas an ordinary desire for safe assets would imply a value closer to zero.

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1 I use the term $r^*$ instead of $r^*$ because $r^*$ has become associated with the New Keynesian model, whereas I make broader structural model assumptions here.
Why this decomposition of $r^\ddagger$?

- Assumptions:
  - log preferences T-period OLG with no discounting
  - fixed capital and no other frictions
- In this type of model, if there was no special desire for safe assets, $r^\ddagger$ would equal the real output growth rate (also the consumption growth rate), $\lambda + \psi$, along the balanced growth path.
- This is one concept of a natural rate of interest.
Longer-run outcomes as regimes

- This conception of the natural real rate of interest suggests $r^*$ will have a constant mean associated with a single possible balanced growth path.
- The point of this presentation is that this single mean may be better modeled as shifting over time.
- Shifting means can be modeled as regime-switching processes.
  - For example, relatively long eras of high productivity growth may be followed by relatively long eras of low productivity growth, and the natural rate of interest would be different in the two regimes.
Data

• I use U.S. data from 1984 to the present.
• I construct an ex-post measure of \( r^+ \) by subtracting the Dallas Fed trimmed-mean PCE inflation rate from the 1-year Treasury rate.¹
• These raw data show a clear downward trend.
• Macroeconomic theory does not like this downward trend—it wants a constant mean.
• Central bank policy may influence this rate over short periods of time, but not over the entire sample period.

¹Forward-looking measures, based on the FRB of Cleveland data, are similar but more volatile.
Real rate of return on short-term government debt, $r^*$

Sources: Federal Reserve Board, FRB of Dallas and author’s calculations. Last observation: September 2017.
The declining trend is on government paper only, not on capital

• The chart shows a declining trend on an ex-post real return to holding government paper.
• The declining trend does not appear to extend to ex-post real returns on claims to capital as measured from the U.S. GDP accounts.
• That return has been fairly constant since the 1980s, as shown in the next chart.
• This provides a rationale for the inclusion of the $\xi$ factor above, which measures the desirability of holding safe assets relative to capital.\(^1\)

\(^1\) For an alternative perspective on this issue, see J.C. Williams, “Three Questions on R-star,” FRB of San Francisco Economic Letter No. 2017-05, Feb. 21, 2017.
Real returns on capital and safe assets

Main question

• Which of the three factors is most important in accounting for this downward trend? Is it productivity growth, labor force growth or the desirability of safe assets?

• I will treat each of these three factors as following a two-state Markov-switching intercept process:

\[ x_t = \mu(s_t) + \varepsilon_t, \]  
where \( \varepsilon_t \) is an i.i.d. error term 

\( s_t \) can take two values, high and low.

• The two possible mean values are called “regimes.”

• The idea is that these types of factors generally have constant means, but that there can be infrequent shifts in mean. I want to characterize these shifts statistically.
Labor Productivity Growth
U.S. labor productivity growth has been low

• A statistical model that estimates the probability that the U.S. economy is in a low-productivity-growth regime puts nearly all the probability on the low-growth regime.\(^1\)

• The most recent estimates from Kahn and Rich (2006) put the growth rate in the low (high) state at 1.24 percent (2.94 percent).\(^2\)

• The U.S. economy was in the high-productivity-growth regime from early 1997 to late 2004.


\(^2\) In previous talks, I have used even lower productivity growth assumptions.
The high- and low-productivity-growth regimes

Labor Force Growth
Labor force growth has been low

• The U.S. labor force had been growing at a 1.33 percent annual rate until the financial crisis.
• The growth rate has been 0.46 percent since the financial crisis.
• It looks like the U.S. is in a low-growth state, but a case could be made that some recent observations have been more consistent with the high-growth state.
• I will consider both possibilities.
The high- and low-labor-force-growth regimes

Investor Desire for Safe Assets
Investor desire for safe assets

• I now remove the regime-switching trends for both labor productivity and labor force growth from the raw data on ex-post safe real returns.
• This leaves us with a time series of adjusted safe real returns, and this series still has a downward trend.
• I then fit a two-state regime-switching process to these adjusted data, and I interpret the two states as a strong desire for safe assets versus a more normal desire for safe assets.
The normal- and high-desire-for-safe-assets regimes

Source: Author’s calculations. Last observation: September 2017.
High-desire-for-safe-assets regime

• The estimated values for $\xi$ are -3.01 percent in the high-desire-for-safe-assets regime and 0.64 percent in the normal-desire-for-safe-assets regime.
• The U.S. is currently in the regime with a high desire for safe assets.
• The difference between the two regimes is largest for this factor; it is in some sense the “most important” of the three.
Current regime: High desire for safe assets

Source: Author’s calculations. Last observation: September 2017.
What Does This Imply for the Natural Real Rate of Interest?
State values for each factor

<table>
<thead>
<tr>
<th>Factor</th>
<th>High state</th>
<th>Low state</th>
<th>High-low state difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor productivity growth, $\lambda$</td>
<td>294</td>
<td>124</td>
<td>170</td>
</tr>
<tr>
<td>Labor force growth, $\psi$</td>
<td>133</td>
<td>46</td>
<td>87</td>
</tr>
<tr>
<td>Investor desire for safe assets (inverse), $\xi$</td>
<td>64</td>
<td>-301</td>
<td>365</td>
</tr>
<tr>
<td>Max/min natural rate, $r^\dagger$</td>
<td>491</td>
<td>-131</td>
<td>622</td>
</tr>
</tbody>
</table>

All values are expressed as basis points. The max (min) natural rate is the value corresponding to all three factors taking the value in the high (low) state.
Using the regime-switching approach

- Labor productivity appears to be in the low-growth regime, so set $\lambda = 1.24$ percent.
- The labor force appears to be in the low-growth regime as well, so set $\psi = 0.46$ percent. Plausibly, labor force growth could be interpreted as switching to the high-growth regime, $\psi = 1.33$ percent.
- There also appears to be a high desire for safe assets, so set $\xi = -3.01$ percent.
- According to this analysis, $r^* = \lambda + \psi + \xi$ is either -131 basis points or -44 basis points, depending on how one views labor force growth.
Recent Related Estimates from the Literature
Related literature and regime switching

• There is a fairly large and growing literature trying to understand the downward trend in the natural rate of interest.
• The literature tends to be quite a bit more sophisticated than the analysis presented here.
• The only point here is to think in terms of regime switching.
• Two of the three factors analyzed, labor productivity growth and the desire for safe assets, are in the low state and do not appear to be shifting to the high state.
• This suggests the natural rate of interest, and hence the Fed’s policy rate, can remain low over the forecast horizon.
Related literature on the natural rate

• Laubach and Williams (2003) impose a structural model and estimate a low $r^*$ without a safe asset demand factor.¹

• Curdia (2015) performs a similar analysis with somewhat altered assumptions and gets a very low $r^*$.²

• Del Negro et al. (2017) impose a structural model, include an evolving demand for safe assets and get a low value for $r^*$.³

• I have imposed less structure along with an alternative stochastic conception, regime switching. This suggests a different view of mean-reversion properties.


Implications for the Policy Rate
The inflation gap

• The U.S. inflation rate has been below the 2 percent inflation target since 2012.*

• Inflation data during 2017 have surprised to the downside and call into question the idea that U.S. inflation is reliably returning toward target.

• Inflation is currently (September 2017) between 37 and 67 basis points below target:
  - Dallas Fed trimmed-mean PCE 1.63%
  - Headline PCE 1.63%
  - Core PCE 1.33%

* The inflation target is in terms of the annual change in the price index for personal consumption expenditures (PCE).
Trimmed-mean PCE inflation lower than expected

U.S. inflation since 2012

The output gap

• I look at three ways to calculate an output gap.
• The CBO output gap (2017-Q3): 0.18 percent
• The deviation from BP(8,32) trend (2017-Q3): 0.22 percent
• Okun’s law implied gap: 0.92 percent
  o St. Louis Fed’s “no-recession regime” estimate: $u^* = 4.5$ percent
  o Unemployment rate (October 2017): $u = 4.1$ percent
  o Output gap: $2.3*(4.5 – 4.1) = 0.92$ percent
Data summary and two policy rules

• I consider two Taylor-type rules:

\[ i = r^* + \pi^e + \phi_\pi \pi^{GAP} + \phi_y y^{GAP} \]

1. Taylor (1993): \( \phi_\pi = 1.5, \phi_y = 0.5 \)
2. Taylor (1999): \( \phi_\pi = 1.5, \phi_y = 1 \)

  o Inflation target: \( \pi^e = \pi^* = 200 \)
  o Natural real rate: \( r^* \in [-131, -44] \)
  o The inflation gap: \( \pi^{GAP} \in [-67, -37] \)
  o The output gap: \( y^{GAP} \in [18, 92] \)

*All values in these calculations are expressed as basis points.*
Policy rate recommendations

• Based on these data and rules, then

• The actual policy rate range today is 100 to 125 basis points, and the federal funds rate is trading at about 116 basis points.
• This is within the range of the recommendations.
Mean-reversion properties

• The regime-switching approach suggests that the current setting of the policy rate is appropriate.
• It also suggests that $r^*$ is unlikely to shift over a forecast horizon of two years (the typical time frame for monetary policy decisions).
• This suggests forward guidance should be characterized by a relatively flat policy rate path, as opposed to an upward-sloping one that would be appropriate if one had a constant $r^*$ with strong mean reversion.
• The median policy rate path in the Summary of Economic Projections (SEP) has had strong mean-reversion assumptions over the last several years.
The FOMC policy rate projections versus reality

Sources: Federal Reserve Board and author’s calculations. Last observation: October 2017.
Conclusion
Conclusions

• This analysis has provided some background on how one might begin to think about recent trends in the natural safe rate of interest in a regime-switching context.

• According to the analysis presented here, the natural rate of interest, and hence the appropriate policy rate, is low and unlikely to change very much over the forecast horizon.

• A more rigorous and thorough analysis that reaches a similar conclusion is Del Negro et al. (2017).
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