Discussion of In-Koo Cho and Kenneth Kasa, “Learning and Model Validation”

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Main idea: Allow agents to select forecasting models in addition to recursive learning.
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Most of the action in actual economies?
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- **Appeal:** Forecasters can “express doubt” about their models, switch models.
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Most of the action in actual economies?

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Mostly thinking in terms of non-nested models.
• Model validation would alter learning dynamics even in economies where REE is unique and expectationally stable.
Connections to escape dynamics

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- Some examples maintain the possibility of escape, others do not.
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Knowledge of the true data generating process is not required. Multiple, misspecified models can be compared.
Speciation testing

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*Dominant recursive learning model* has smallest asymptotic rejection probability.

Main result: The authors provide conditions under which validation dynamics converge to the dominant recursive learning model, which the agent then uses almost always.
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- In this paper, agents are endowed with multiple, simple, misspecified PLMs. Agents use endogenously generated data to try to find the “best choice” among these, the least falsifiable.

- Each model may induce a self-confirming equilibrium.
Nature of the validation dynamics

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- Asynchronous updating (p. 12). Agent may be unaware that an alternative model is better until rejection of current model occurs.
Instability generated by rival models

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- A general difficulty for validation dynamics is that models are being fit to data generated by other models. A policymaker that is learning would know this.
• The ambitious goals reminiscent of economic applications of classifier systems and genetic algorithm learning from the artificial intelligence literature:
Discussion of Cho and Kasa, “Validation.”

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Discussion
Parameters versus models
Connections to escape dynamics
Model validation
Specification testing
Assignment of the PLM
Nature of the validation dynamics
Instability generated by rival models

Artificial intelligence
Comparison with artificial intelligence
Statistical versus economic selection
Hypothesis testing
Restricted perceptions example
Conquest example
Conclusions

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- Evolutionary dynamic not part of the story here.
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Cogley-Sargent (2004) story about Samuelson-Solow vs. Lucas-Sargent. The decision-maker downweights the evidence because of the economic consequences of choosing the wrong model.

Kocherlakota (2006): better fit not the same as better model.
• Model $k$ remains the model of choice for an extended period.
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• What economic advantage does the agent gain from resistance to switching models?
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• Why not simply adopt today’s best model?
Choice between two misspecified models is made based on

\[ H_1 (\beta_1) = \left( \frac{2 (1 - \alpha)}{\eta \Sigma_1 (\bar{\beta}_1)} \right) (\beta_1 - \bar{\beta}_1)^2 \]  

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\[ H_2 (\beta_2) = \left( \frac{2 (1 - \alpha)}{\eta \Sigma_2 (\bar{\beta}_2)} \right) (\beta_2 - \bar{\beta}_2)^2 \]  

(2)

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But this may not be the better restricted perceptions equilibrium for household allocations.

The agent may prefer to use an alternative model, experience the RPE associated with that model, and adopt that.
• Parsimony favors the static reference model. Switch to it near the SCE.
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The switch to the static model is reminiscent of Brock and Hommes (1997).
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• The switch to the static model is reminiscent of Brock and Hommes (1997).

• Parsimony a key ingredient in this story. Something to hang our hats on?
• A fascinating paper.
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Doing more than a ‘minimal deviation from rational expectations’?