Demographics, Redistribution, and Optimal Inflation

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Demographic Changes and Macroeconomic Performance

The views expressed herein do not necessarily reflect those of the FOMC or the Federal Reserve System.
Inflation and Demography

- Can observed low inflation outcomes be related to demographic factors such as an aging population?
- A basic “back-of-the-envelop” suggests NO
- Consider an economy were capital and money are perfect substitutes
  \[ r = \delta + n = \frac{1}{1 + \pi} \]
- The effect of a permanent increase in \( n' > n \) increases the return of capital \( r' > r \) and inflation decreases to \( \pi' \).
- Countries with relatively young (old) populations would have relatively low (high) inflation rates, all else equal.
Objective

- Understand the determination of central bank objectives when population aging shifts the social preferences for redistribution and its implications for inflation.
- The intergenerational redistribution tension is intrinsic in life-cycle models.
  - Young cohorts do not have any assets and wages are the main source of income.
  - Old generations cannot work and prefer a high rate of return from their savings.
- When the old have more (less) influence over redistributive policy, the rate of return of money is high (low).
Objective

- Use a direct mechanism to decide the allocations. A baby boom corresponds to putting more weight on the young of a particular generation relative to past and future generations.
- This mechanism can replicate any *steady state* allocation arising from a political economy model with population growth or decline.
Outline Presentation

- Efficient economy and intergenerational redistribution
- Constrained efficiency and redistribution
- Optimal Wedges: Capital taxes=Inflation
- Numerical examples
  - Transitory demographics
  - Persistent demographic changes
Model
Two-period OLG model with capital.

Discrete time $t = ..., -2, -1, 0, 1, 2, ...$

Population growth $N_t = (1 + n)N_{t-1}$ where $N_0 = 1$

Preferences: $U(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1})$

Neoclassical production $F(K_t, N_t)$ and constant depreciation $\delta$

Per capital resource constraint

$$c_{1,t} + \frac{1}{1+n}c_{1,t-1} + (1 + n)k_{t+1} = f(k_t) + (1 - \delta)k_t.$$
The objective function weights current and future generations according to

\[ V(k_0) = \max \{ \beta \lambda_{-1} u(c_{2,0}) + \sum_{t=0}^{\infty} \lambda_t [u(c_{1,t}) + \beta u(c_{2,t+1})] \} \]

subject to the resource constraint.

Optimality conditions imply

\[ \frac{u'(c_{1,t})}{u'(c_{2,t})} = \frac{\lambda_{t-1}}{\lambda_t} \beta (1 + n) \]

and

\[ (1 + n) \frac{u'(c_{1,t})}{u'(c_{1,t+1})} = \frac{\lambda_{t+1}}{\lambda_t} \left[ 1 - \delta + f'(k_{t+1}) \right]. \]
Efficient production: the steady state stock of capital $k^s$ is determined by

$$f'(k^s) = (1 + n)\lambda^{-1} + \delta - 1,$$

For $\lambda < 1$, the economy is dynamically efficient. When $\lambda = 1$, the economy satisfies the golden rule $f'(k^*) = n + \delta$.

Efficient consumption $c_1^s$ and $c_2^s$ solve

$$u'(c_1^s) = \beta(1 + n)u'(c_2^s)$$

$$c_1^s + \frac{c_2^s}{1 + n} + (\delta + n)k^s = f(k^s).$$
Consumers: Representative newborn solves

\[ \max u(c_{1,t}) + \beta u(c_{2,t+1}) \]

s.t. \[ c_{1,t} + s_t = w_t l_t + T_{1,t}, \]
\[ c_{2,t+1} = (1 - \delta + r_{t+1}) s_t + T_{2,t+1}. \]

The optimality condition

\[ u'(w_t l_t - s_t + T_{1,t}) = \beta u' [(1 - \delta + r_{t+1}) s_t + T_{2,t+1}] (1 + r_{t+1}). \]

Intergenerational redistribution:

\[ T_{1,t} + \frac{T_{2,t}}{1+n} = 0. \]
No Intergenerational Redistribution (Ramsey)

- In the absence of intergenerational redistribution

\[ V(k_0) = \max \{ \beta \lambda_{-1} u(c_{2,0}) + \sum_{t=0}^{\infty} \lambda_t [u(c_{1,t}) + \beta u(c_{2,t+1})] \} \]

s.t. \[ c_{1,t} = f_l(k_t) l - (1 + n) k_{t+1}, \]
\[ c_{2,t} = [1 - \delta + f_k(k_t)] k_t, \]

- Optimality conditions (endogenous multipliers \( \gamma_{1,t}, \gamma_{2,t} \))

\[ \frac{u'(c_{1,t})}{u'(c_{2,t})} = \frac{\lambda_{t-1}}{\lambda_t} \beta (1 + n) \frac{\gamma_{1,t}}{\gamma_{2,t}} \]
Markets Redistribute: Capital taxes/inflation

- The intergenerational decision of savings (capital) is more complicated:

\[
(1 + n) u'(c_{1,t}) = \frac{\lambda_{t+1}}{\lambda_t} u'(c_{1,t+1}) f_{l,k} \frac{l}{1 + n}
\]

\[\uparrow\text{savings}\]

\[\uparrow\text{wage rate}\]

\[+ \beta u'(c_{2,t+1}) [1 - \delta + f_k + f_{k,k} \frac{s_t}{1 + n}]\]

\[\downarrow\text{return all savings}\]

- Efficient wedges

\[
\frac{u'(c_{1,t})}{\beta u'(c_{2,t+1})} = \frac{\left[1 - \delta + f_k \left(\frac{s_{t-1}}{1 + n}\right) (1 + \phi_k^t)\right]}{(1 + \phi_{t+1}^\lambda)}.
\]

where

\[
\phi_k^t = \frac{kf_{k,k}}{f_k} < 1; \quad \phi_{t+1}^\lambda = \lambda \frac{u'(c_{1,t+1})}{u'(c_{1,t})} f_{k,k} \left(\frac{s_{t-1}}{1 + n}\right) \frac{l}{1 + n} < 1
\]
The optimal intergenerational redistribution determines the equilibrium interest rate.

These parameters determine inflation when capital and money are perfect substitutes.

Per capita money growth rate evolves

\[ M_{t+1} (1 + n) = (1 + z_t) M_t \]

Arbitrage then implies that

\[ f_k(k_t) = \frac{1}{1 + \pi_t} = \frac{1 + n}{1 + z_t}. \]

Money is priced as an asset that is held in zero net supply (Woodford’s (2003) “cashless” economy).
Quantitative Illustration
Preferences:

\[ U(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-\sigma}}{1 - \sigma} + \beta \frac{c_{2,t+1}^{1-\sigma}}{1 - \sigma}, \]

Technology:

\[ f(k) = Ak^\alpha \]
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<th>Parameter</th>
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Steady State
Intergenerational Discounting ($\lambda$)

Capital Stock

- Efficient
- Constrained
Inflation ($n<0$)
Inflation (n=0)
Transitional Dynamics: Demographics and Inflation
Intergenerational Redistribution: Transitory
Interest Rates and Demographics: Transitory

- Efficient
- Constrained
Intergenerational Redistribution: Permanent

Intergenerational Discounting ($\lambda$)
Interest Rates and Demographics: Permanent

![Graph showing interest rate changes with efficient and constrained scenarios.](image-url)
Conclusions

- Study the interaction between population demographics, the desire for redistribution in the economy, and the optimal inflation rate.

- The intergenerational redistribution tension is intrinsic in life-cycle models.
  - Young cohorts do not have any assets and wages are the main source of income.
  - Old generations cannot work and prefer a high rate of return from their savings.

- When the old have more (less) influence over redistributive policy, the rate of return of money is high (high).