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# An Illustrative Calculation of $r^*$

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*Any opinions expressed here are my own and do not necessarily reflect those of the Federal Open Market Committee.*

# Introduction

# Why worry about $r^*$ ?

- U.S. unemployment and inflation gaps are approximately zero.
- In this situation, a Taylor-type rule collapses to a Fisher equation:

$$\begin{aligned}i &= r^* + \pi^e + \phi_\pi \pi^{GAP} + \phi_u u^{GAP} \\ &= r^* + \pi^e\end{aligned}$$

where  $\pi^e = 2$  percent, the FOMC's inflation target.

- Bottom line: With the gaps near zero, a Taylor-type rule simply recommends setting the policy rate equal to the value of  $r^*$  plus 2 percent.
- But what is the value of  $r^*$ ?

# Decomposing $r^\dagger$

- $r^\dagger$  is often referred to as “the natural real rate of interest.”<sup>1</sup>
- One way to think of it is to divide it into three factors:

$$r^\dagger = \lambda + \psi + \xi, \text{ where}$$

- $\lambda$ : the labor productivity growth rate
- $\psi$ : the labor force growth rate
- $\xi$ : an investor desire for safe assets. A strong desire for safe assets would imply a relatively large negative value for  $\xi$ , whereas an ordinary desire for safe assets would imply a value closer to zero.

<sup>1</sup> I use the term  $r^\dagger$  instead of  $r^*$  because  $r^*$  has become associated with the New Keynesian model, whereas I make broader structural model assumptions here.

# Why this decomposition of $r^*$ ?

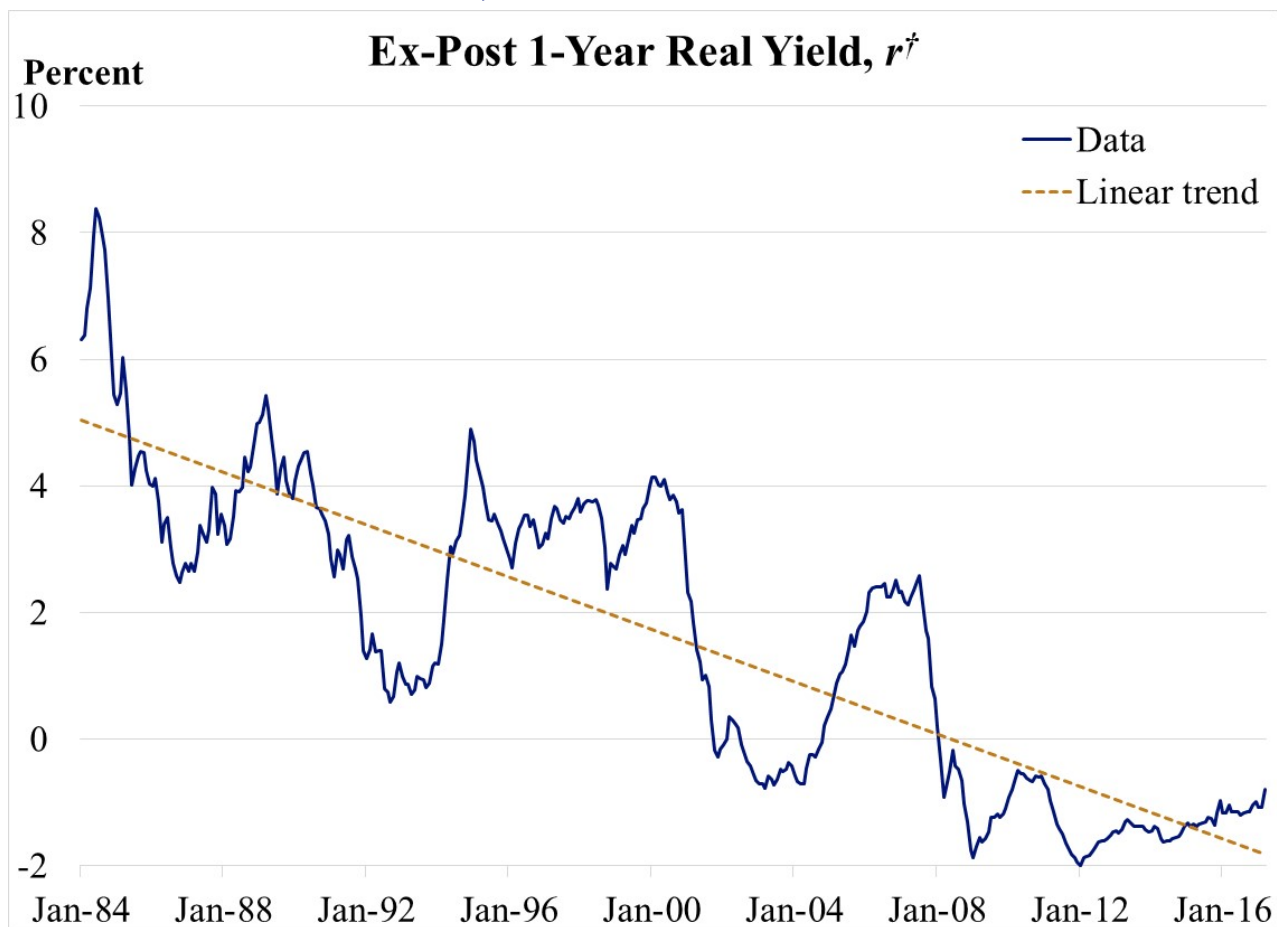
- Assumptions:
  - log preferences T-period OLG with no discounting
  - fixed capital and no other frictions
- In this type of model, if there was no special desire for safe assets,  $r^*$  would equal the real output growth rate (also the consumption growth rate) along the balanced growth path.
- This is one concept of a natural rate of interest.
- This concept suggests  $r^*$  will have a constant mean, but the point of this presentation is that this mean may shift over time.

# Data

- I use U.S. data from 1984 to the present.
- I construct an ex-post measure of  $r^*$  by subtracting the Dallas Fed trimmed-mean PCE inflation rate from the 1-year Treasury rate.<sup>1</sup>
- These raw data show a clear downward trend.
- Macroeconomic theory does not like this downward trend—it wants a constant mean.

<sup>1</sup>Forward-looking measures, based on the FRB of Cleveland data, are similar but more volatile.

# Real rate of return on short-term government debt, $r^*$



Source: Federal Reserve Board, FRB of Dallas and author's calculations. Last observation: March 2017.

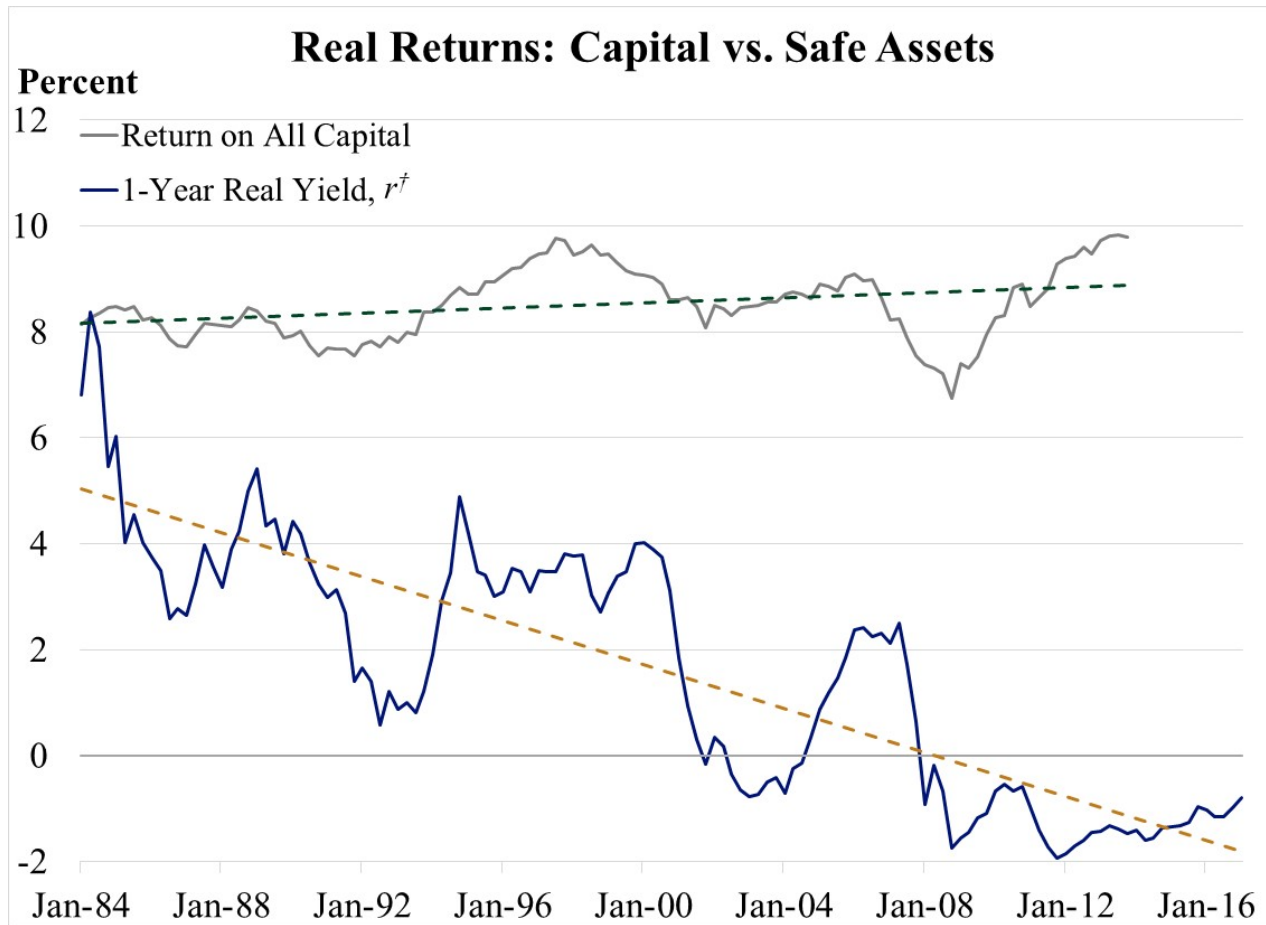
# The declining trend is on government paper only, not on capital

- The chart shows a declining trend on an ex-post real return to holding government paper.
- The declining trend does not appear to extend to ex-post real returns on claims to capital as measured from the U.S. GDP accounts.
- That return has been fairly constant since the 1980s, as shown in the next chart.
- This provides a rationale for the inclusion of the  $\xi$  factor above, which measures the desirability of holding safe assets relative to capital.<sup>1</sup>

<sup>1</sup> For an alternative perspective on this issue, see J.C. Williams, “Three Questions on R-star,” FRB of San Francisco Economic Letter No. 2017-05, Feb. 21, 2017.



# Real returns on capital and safe assets



Source: P. Gomme, B. Ravikumar and P. Rupert. "Secular Stagnation and Returns on Capital," FRB of St. Louis Economic Synopses No. 19, 2015; Federal Reserve Board, FRB of Dallas and author's calculations. Last observation: 2017-Q1.

# Main question

- Which of the three factors is most important in accounting for this downward trend? Is it productivity growth, labor force growth or the desirability of safe assets?
- I will treat each of these three factors as following a two-state Markov switching intercept process:

$$x_t = x(s_t) + \varepsilon_t, \text{ where } \varepsilon_t \text{ is an i.i.d. error term}$$

$s_t$  can take two values, high and low.

- The two possible mean values are called “regimes.”
- The idea is that these types of factors generally have constant means, but that there can be infrequent shifts in mean. I want to characterize these shifts statistically.

# Labor Productivity Growth

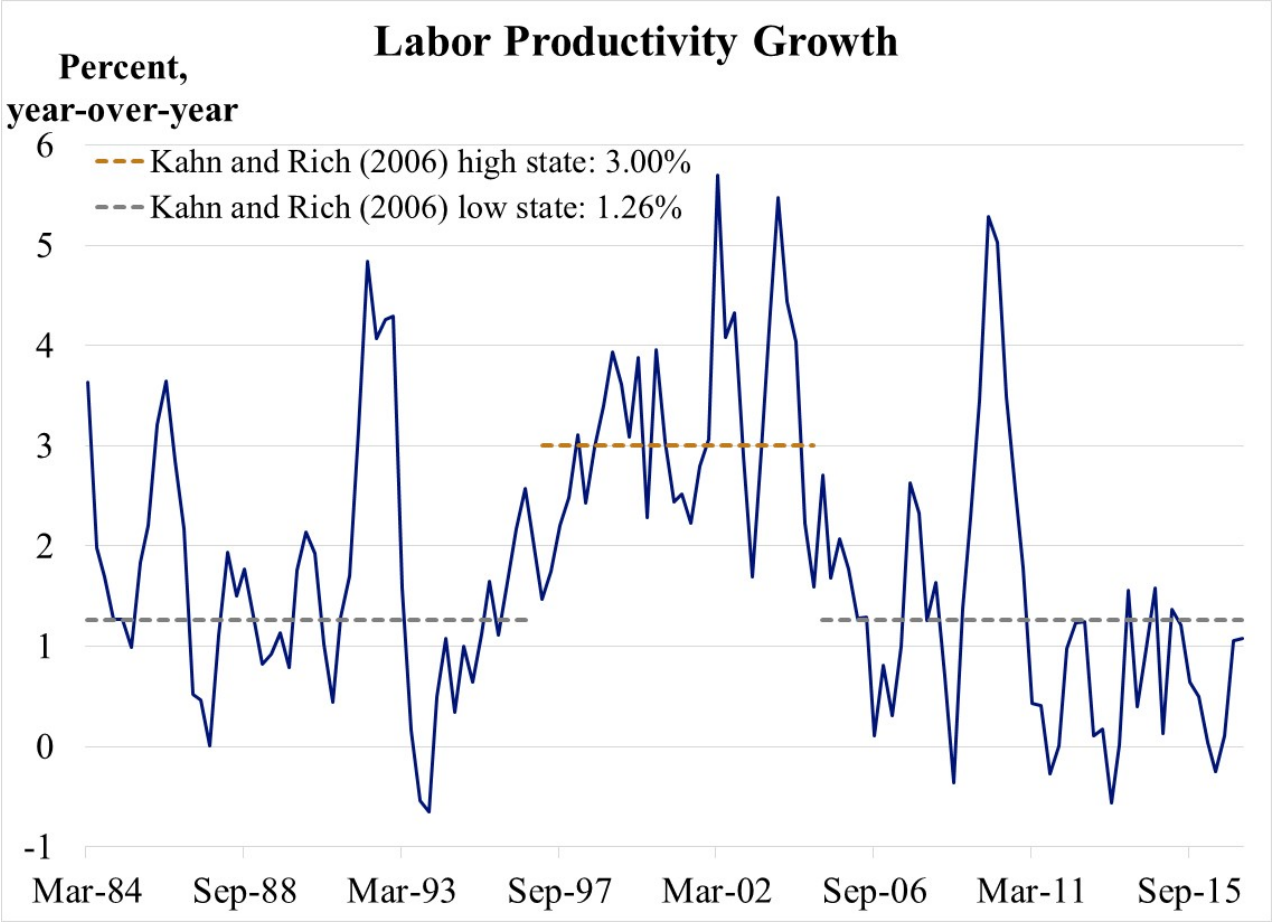
# U.S. labor productivity growth has been low

- A statistical model that estimates the probability that the U.S. economy is in a low-productivity-growth regime puts nearly all the probability on the low-growth regime.<sup>1</sup>
- The most recent estimates from Kahn and Rich (2006) put the growth rate in the low (high) state at 1.26 percent (3.0 percent).<sup>2</sup>
- The U.S. economy was in the high-productivity-growth regime from early 1997 to late 2004.

<sup>1</sup> See J.A. Kahn and R.W. Rich, 2006, "Tracking Productivity in Real Time," Federal Reserve Bank of New York, Current Issues in Economics and Finance, 12(8).

<sup>2</sup> In recent talks, I have used even lower productivity growth assumptions.

# The high- and low-productivity-growth regimes



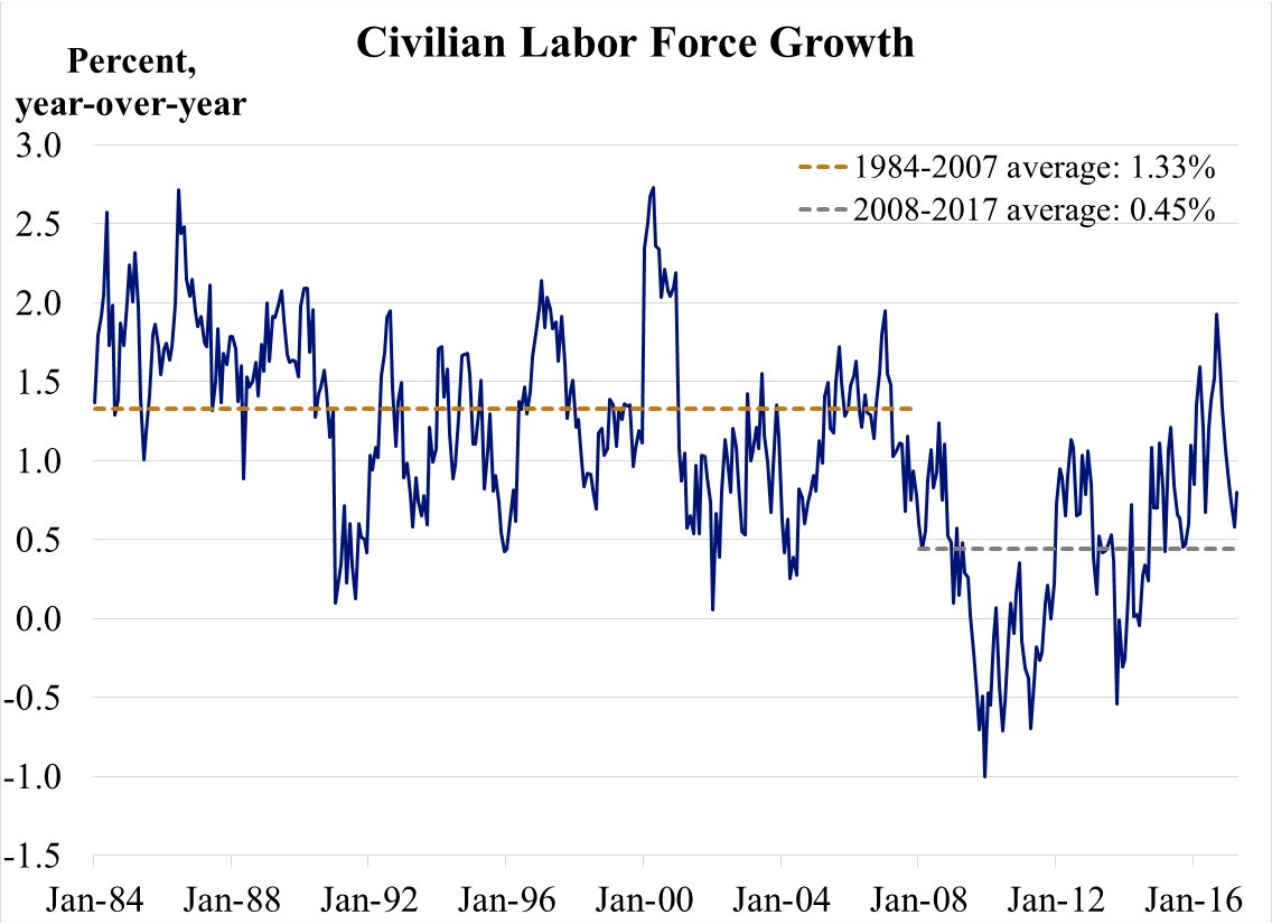
Source: Kahn and Rich (2006) and FRB of New York. Last observation: 2017-Q1.

# Labor Force Growth

# Labor force growth has been low

- The U.S. labor force had been growing at a 1.33 percent annual rate until the Great Recession.
- The growth rate has been 0.45 percent since the Great Recession.
- It looks like the U.S. is in a low-growth state, but a case could be made that some recent observations have been more consistent with the high-growth state.
- I will consider both possibilities.

# The high- and low-labor-force-growth regimes



Source: Bureau of Labor Statistics and author's calculations. Last observation: April 2017.

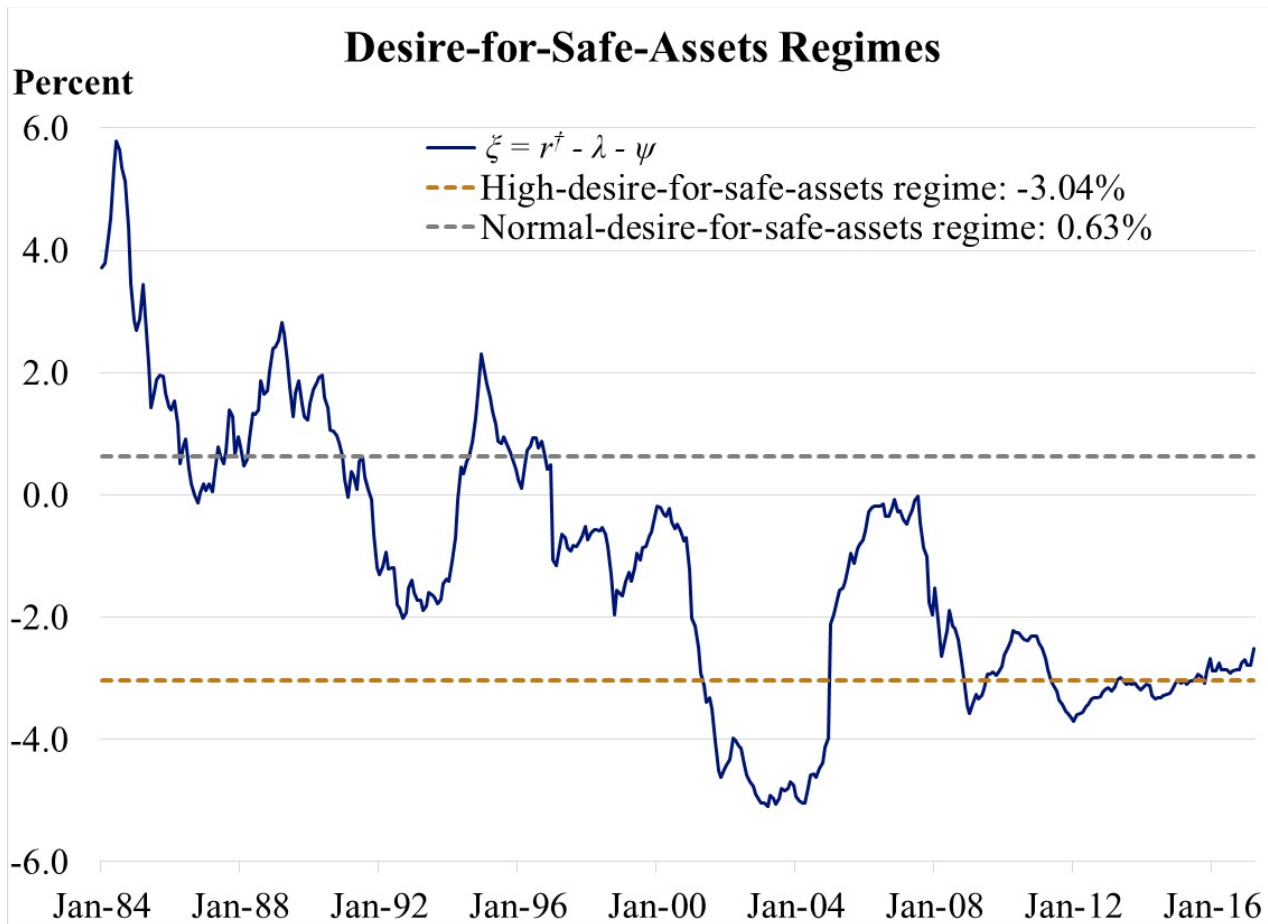


# Investor Desire for Safe Assets

# Investor desire for safe assets

- I now remove the regime-switching trends for both labor productivity and labor force growth from the raw data on ex-post safe real returns.
- This leaves us with a time series of adjusted safe real returns, and this series still has a downward trend.
- I then fit a two-state regime-switching process to this adjusted data, and interpret the two states as a strong desire for safe assets versus a more normal desire for safe assets.

# The normal- and high-desire-for-safe-assets regimes

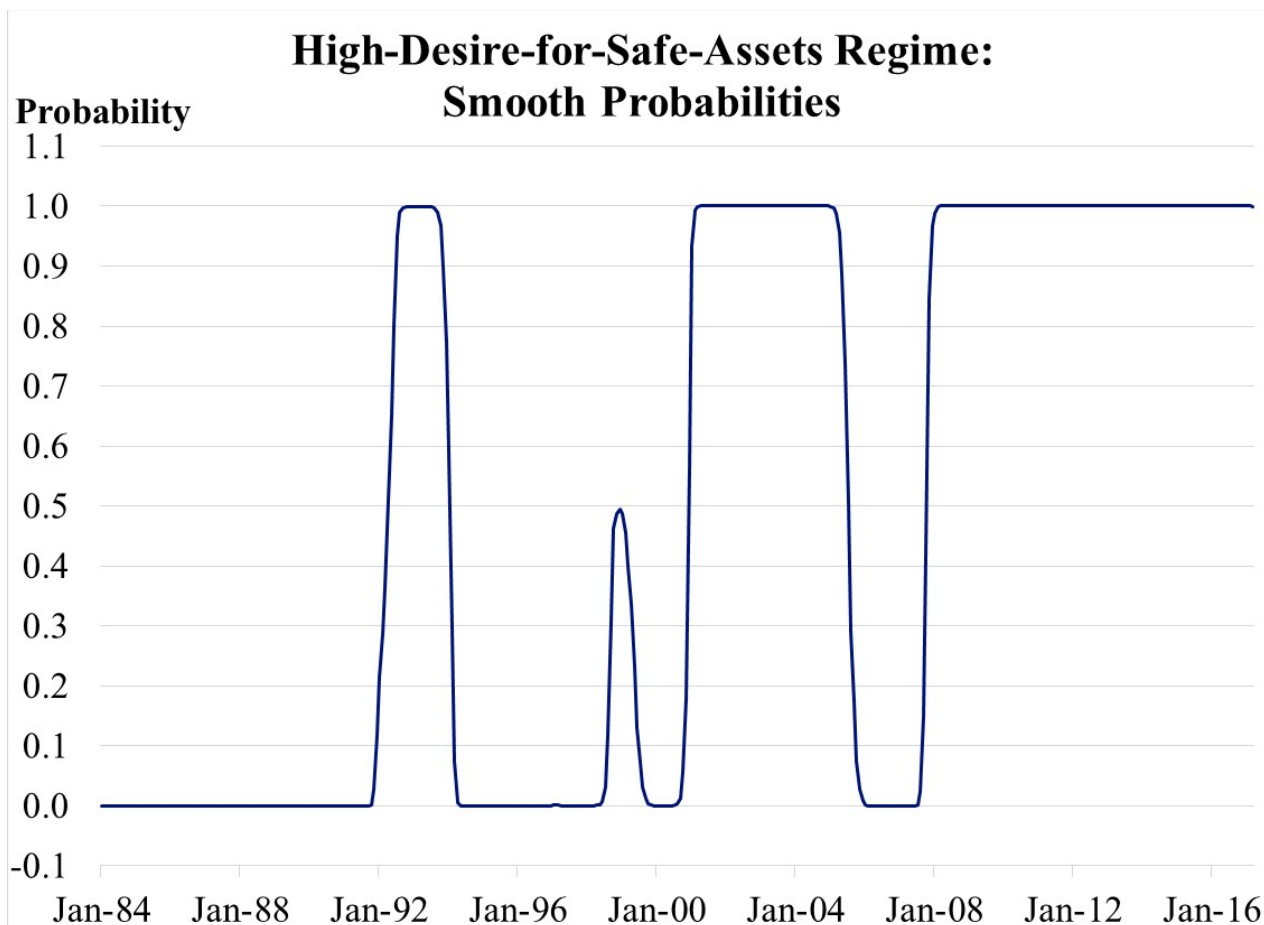


Source: Author's calculations. Last observation: March 2017.

# High-desire-for-safe-assets regime

- The estimated values for  $\xi$  are -3.04 percent in the high-desire-for-safe-assets regime and 0.63 percent in the normal-desire-for-safe-assets regime.
- The U.S. is currently in the regime with a high desire for safe assets.
- The difference between the two regimes is largest for this factor; it is in some sense the “most important” of the three.

# Current regime: High desire for safe assets



*Source: Author's calculations. Last observation: March 2017.*

# What Does This Imply for the Natural Real Rate of Interest?

## State values for each factor

Factor	High state	Low state	High-low state difference
Labor productivity growth, $\lambda$	300	126	174
Labor force growth, $\psi$	133	45	88
Investor desire for safe assets (inverse), $\xi$	63	-304	367
<b>Max/min natural rate, <math>r^{\dagger}</math></b>	<b>496</b>	<b>-133</b>	<b>629</b>

*All values are expressed as basis points. The max (min) natural rate is the value corresponding to all three factors taking the value in the high (low) state.*

# Using the regime-switching approach

- Labor productivity appears to be in the low-growth regime, so set  $\lambda = 1.26$  percent.
- The labor force appears to be in the low-growth regime as well, so set  $\psi = 0.45$  percent. Plausibly, labor force growth could be interpreted as switching to the high-growth regime,  $\psi = 1.33$  percent.
- There also appears to be a high desire for safe assets, so set  $\xi = -3.04$  percent.



# Implications for the policy rate

- According to this analysis,  $r^* = \lambda + \psi + \xi$  is either -133 basis points or -45 basis points, depending on how one views labor force growth.
- If I add 200 basis points to account for the inflation target, I get an appropriate policy rate setting from a Taylor-type rule of either 67 basis points or 155 basis points.
- The actual current policy rate is about 88 basis points.
- The policy rate is approximately at an appropriate setting today according to this analysis and with gap variables assumed to be zero.

# Recent Related Estimates from the Literature

# Related literature and regime switching

- There is a fairly large and growing literature trying to understand the downward trend in the natural rate of interest.
- The literature tends to be quite a bit more sophisticated than the analysis presented here.
- The only point here is to think in terms of regime switching.
- Two of the three factors analyzed, labor productivity growth and the desire for safe assets, are in the low state and do not appear to be shifting to the high state.
- This suggests the natural rate of interest, and hence the Fed's policy rate, can remain low over the forecast horizon.

# Related literature on the natural rate

- Laubach and Williams (2003) impose a structural model and estimate a low  $r^*$  without a safe asset demand factor.<sup>1</sup>
- Curdia (2015) performs a similar analysis with somewhat altered assumptions and gets a very low  $r^*$ .<sup>2</sup>
- Del Negro et al. (2017) impose a structural model, include an evolving demand for safe assets and get a low value for  $r^*$ .<sup>3</sup>
- I have imposed less structure along with an alternative stochastic conception, regime switching. This suggests a different view of mean-reversion properties.

<sup>1</sup> T. Laubach and J.C. Williams, “Measuring the Natural Rate of Interest,” *Review of Economics and Statistics*, November 2003, 85(4), 1063–70.

<sup>2</sup> V. Curdia, “Why So Slow? A Gradual Return for Interest Rates,” *FRB of San Francisco Economic Letter No. 2015-32*, Oct. 12, 2015.

<sup>3</sup> M. Del Negro, D. Giannone, M.P. Giannoni and A. Tambalotti, “Safety, Liquidity and the Natural Rate of Interest,” *Brookings Papers on Economic Activity*, Spring 2017, conference draft.

# Conclusion

# Conclusions

- This analysis has provided some background on how one might begin to think about recent trends in the natural safe rate of interest in a regime-switching context.
- According to the analysis presented here, the natural rate of interest, and hence the appropriate policy rate, is low and unlikely to change very much over the forecast horizon.
- A more rigorous and thorough analysis that reaches a similar conclusion is Del Negro et al. (2017).



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