Fractile vs. Equal

GEOFRED
GEOGRAPHICAL **ECONOMIC DATA** | ST. LOUIS FED

Lesson Description

Students work with data that represent the ages of 24 people to learn the difference between categorizing data in fractile intervals and equal intervals. Students discuss dividing bonus points among class members to understand what per capita means. Then students look at per capita personal income by state using the GeoFRED™ mapping tool. They compare per capita personal income displayed with data in equal intervals and with data in fractile intervals.

Grade Level

Grades 7-10

Concepts

Equal intervals
Fractile intervals
Income
Per capita personal income

Objectives

Students will:

■   Explain the differences between equal intervals and fractile intervals.

■   Define personal income and per capita personal income.

■   Compare tables and maps of the same data displayed with equal intervals and with fractile intervals.

Content Standards

National Standards in Economics

■   **Standard 13:** Income for most people is determined by the market value of the productive resources they sell. What workers earn depends, primarily, on the market value of what they produce and how productive they are.

•   Benchmark 2, Grade 8: To earn income, people sell productive resources. These include their labor, capital, natural resources and entrepreneurial talents.

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National Geography Standards

Element One: The World in Spatial Terms

■ **Standard 1**: Globes and maps have been among the most ubiquitous tools for learning geography. They have been joined by aerial photographs, remotely sensed images, and geographic information systems. As technology makes them easier to make, maps and other geographic representations appear practically everywhere.

Time Required

45-60 minutes

Materials

■ Visuals of Handouts 1 and 2

■ A copy of Handout 1 for each pair of students

■ A copy of Handouts 2 and 3 for each student

■ Scissors for each pair of students

■ A calculator for each student

Procedures

1. Explain that a marketing research firm interviewed several people at a large suburban mall regarding their shopping habits. During the interviews, they asked each participant to provide his or her age.

2. Display a visual of *Handout 1: Age Cards* and explain that this visual lists the ages of one group of participants that was interviewed. Explain that students are going to use the age information to learn about ways to organize data to make it easy to understand and analyze. Point out that there are at least two ways to organize this list of ages or set of data. One way is to establish categories or age-range values and to then place each age into the appropriate category—for example, 20-25 years, 26-31 years and so on. Each category has the same age range, i.e. six years.

   Another possibility is to place an equal, or nearly equal, number of data points/ages in each category and to then build the ranges accordingly. For example, suppose there are five people in the 16-19 age range, five people in the 20-30 range, five people in the 31-39 range, etc. In this case, the range for the categories varies, but the quantity of ages in each category is the same for all categories.

3. Divide the students into pairs. Distribute a copy of *Handout 1: Age Cards* and a pair of scissors to each pair of students. Tell half of the pairs in the room to cut out their cards and organize the data into categories, with each category having the same number of age cards/data points. Tell these student pairs to write the value of the beginning and ending range for each category.

   For example, if each range is supposed to have three data points/ages in it, then one category might be 15-19 years. With this type of categorizing, the range in number of years per category might vary from category to category, but there will be the same number of data points, or cards, in each category. Point out that the group- ings must have consecutive ages—that is 15-19, 20-27, etc., vs. 15, 20, 27, etc. Point out that there are different possible answers.

4. Tell pairs in the other half of the class to cut the cards apart and divide the cards into categories of equal range. Tell them to write the range for each category, such as 20-25 years and so on. Remind them that each age-range/category should be equal—that is, it should include the same number of years. For example, a category titled "20-27 years" would include eight ages (20, 21, 22, 23, 24, 25, 26, 27) and therefore other ranges should include eight ages, too. Point out that there are dif- ferent possible answers.

5. Allow time for students to work. Ask several pairs from each half of the class to share their work.

6. Display *Handout 2: Equal vs. Fractile,* refer to Table A, and review the categories/ intervals in column 1. Ask the students if the age range (number of ages) for each category is the same. *(Yes, five ages.)*

7. Distribute a copy of Handout 2 to each student and have students look at Table A. Review the intervals in column 1 and organize their cards into those intervals. Discuss the following:

   • The first interval is 15-19 years.

   • How many age cards fall in this interval? *(three—16, 17 and 19)*

   • Write these numbers in the second column of Table A across from the 15-19 year interval.

   • Continue with similar discussions for each interval in Table A. Refer to *Handout 2: Equal vs. Fractile—Answer Key* to check student work and record information on the chart.

8. Refer to Table B on the visual of Handout 2 and review the intervals in column 3. Instruct student pairs to organize their cards into these intervals. Ask the students to report the number of age cards in each category. Write that information in col- umn 4 across from each interval. Refer to *Handout 2: Equal vs. Fractile—Answer Key.* Discuss the following:

- How do the age intervals differ in Table A and Table B? *(In Table A, column 1, each of the age intervals are the same size. In Table B, column 3, the age intervals are of different sizes.)*

- How do the number of data points in each age interval differ in Table A and in Table B? *(In Table A, there is a different number of data points in the age intervals. In Table B, there is the same number of data points in each age interval.)*

9. Explain that when data are divided into equal-size, ranges or intervals, the method of categorizing the data is called **equal interval**. In the first row of Table A, next to the words "Table A," write the title "Equal Interval." Explain that when data are divided into intervals so that there is the same number, or nearly the same number, of data points in each interval, the method of categorizing the data is called **fractile interval**. In the first row of Table B, next to the words "Table B," write the title "Fractile Interval."

10. Ask students where they would have placed one additional card representing a 20-year-old person. Explain that the additional 20-year-old would have fallen into the "15-20 years" interval, giving that interval five data points.

11. Point out that with both fractile interval and equal interval, the set of cards or data points were the same, but the way in which the data were organized was different. Point out that neither way of organizing the data—equal interval or fractile interval—is wrong or right. But the organization may give those reviewing the data different ideas about the data and allow people to tell slightly different stories about the data.

12. Tell the students that they are going to look at some data in GeoFRED. Explain that GeoFRED is an internet tool that displays data on maps. In this case, students will be looking at per capita personal income data. Define "per capita" as per person. To help students understand the idea of per capita, use the following example: *(Note: For this example, determine the total number of bonus points available by multiplying the number of students in your class by 10.)*

   - Over the semester so far, students have earned a total of 240 bonus points.

   - To determine how the bonus points relate to the number of students in the class, we need to determine per capita bonus points.

   - The same number of points will be available for each student in the class. How many points will be available per student? *(number of points ÷ number of students = 10 points)*

13. Open an internet link and enter the address: http://geofred.stlouisfed.org/. Tell students that this is the GeoFRED site of the Federal Reserve Bank of St. Louis. GeoFRED is designed to map data; that is, to take data about countries and show the data on a map.

14. Tell students that one set of data available on GeoFRED is per capita personal income by state. Remind the students that per capita means per person. Explain that **income** is the payment people earn for the work that they do. **Per capita personal income** is the total income earned by individuals in a state, region, or country during a year, divided by the population of the state, region, or country. For this map, we will use per capita personal income by state.

15. Click on the arrow at the top of the GeoFRED screen. Click on "Tools" in the upper-left corner. From the "Choose Data" menu, select as follows:

    • Region Type: State

    • Data: Per Capital Personal Income (Not Seasonally Adjusted, Annual, Dollars)

    • Frequency: Annual

    • Units: Dollars

    • Date: 2011

    • Click on "Edit Legend." For "Interval Method," choose "Equal." Close the "Tools" tab.

16. Move the cursor over each state to see that state's per capital personal income. Display per capita personal income on each state by using the scale on the upper right to zoom in. Refer students to the legend/key for the map. Point out that the data are reported in dollars. Discuss the following.

    • Into how many intervals are the data divided? *(five)*

    • Are the intervals equal? *(Yes, each interval is $8,360.)*

    • Which color represents the highest category of per capita personal income? *(the darkest color)*

    • Which color represents the lowest category of per capita personal income? *(the lightest color)*

    • How many states are in the lowest category in 2011? *(26—Alabama, Arizona, Arkansas, Florida, Georgia, Idaho, Indiana, Kentucky, Louisiana, Maine, Michigan, Mississippi, Missouri, Montana, Nevada, New Mexico, North Carolina, Ohio, Oklahoma, Oregon, South Carolina, Tennessee, Texas, Utah, West Virginia, Wisconsin)*

    • Zoom in the Northeast region and ask which state has the highest per capita personal income in this region. *(Connecticut)*

    • Does Connecticut have the highest value on the legend? *(No.)*

    • Which state has a per capita personal income of $73,783? *(Allow students to guess why they are not able to locate the state with this level of personal income. Zoom in around the Washington, D.C. area to reveal that the highest per capita personal income is earned in the District of Columbia.)*

    • Knowing that the intervals are equal in size and that the number of data points in each interval vary, is the data categorized into equal intervals or fractile intervals? *(equal intervals)*

- Would you describe this map as being primarily light-colored or primarily dark-colored? *(primarily light-colored)*

- If the map is primarily light-colored, what might you summarize about state-level per capita incomes in the United States in 2011? *(Answers may vary. Some students might say that per capita incomes in the United States are relatively low and homogeneous or similar.)*

17. Explain that GeoFRED also allows users to organize data in fractile categories. Ask students what that means. (Sizes of the interval may vary, but the number of data points—in this case states—in each interval will be equal.)

18. Click on "Tools" and "Edit Legend." Change "Interval Method" to "Fractile."

    Discuss the following:

    - Which colors represents higher per capita income? *(the darker colors)*

    - Which color represents lower per capita incomes? *(the lighter colors)*

    - Referring to the table showing in the left menu, what is the interval for each category? *(category 1: $3,689; category 2: $2,280; category 3: $4,481; category 4: $3,431; category 5: $27,906)*

    - What would explain the variance between the differences in the first four categories and the difference between the 4th and 5th category? *(the high level of per capita personal income in Washington, D.C.)*

    - What is the second highest level of per capita personal income among the states, and to which state is it attributed? *($57,902; Connecticut)*

    - If Washington, D.C. were eliminated from the list, what would be the interval between the 4th and 5th categories? *($12,021)*

    - How many states are in the lowest category of personal income, and which states are included? *(11; Alabama, Arizona, Arkansas, Idaho, Indiana, Kentucky, Mississippi, New Mexico, South Carolina, Utah, West Virginia)*

    - How many states are in the highest category of personal income, and which states are included? *(9; Connecticut, Maryland, Massachusetts, New Jersey, New York, North Dakota, Virginia, Wyoming, and Washington, D.C.)*

    - What is different about the number of data points (states) in each interval for this map? *(the number of states in each interval is nearly the same)*

    - What is different about the intervals for this map? *(no longer equal)*

    - What might you summarize about state-level per capita income in the United States in 2011 from looking at this map? *(Answers will vary but might include that there are more differences in income and less homogeneity than when the data are categorized in equal intervals.)*

19. Point out that if people do not carefully review the keys for these two maps *(equal interval and fractile interval)*, the information might be misinterpreted. It is important that people ask questions about data that is presented to them, such as how the data are organized and why the data are organized in that way.

Closure

20. Review important content by asking the following questions:

   • How does categorizing data in equal intervals differ from categorizing data in fractile intervals? (With equal categorization, intervals or ranges of the same size are established and data are categorized in those ranges. There will be varying number of data points in each range. With fractile categorization, intervals or ranges are established so that there is an equal number, or nearly equal number, of data points in each range or interval.)

   • What does per capita mean? (per person)

   • What is personal income? (The payment people earn for the work that they do)

   • What is per capita personal income? (the total income earned by people in a state, region, or country, divided by the population of the state, region, or country)

Assessment

21. Distribute a copy of *Handout 3: Assessment* to each student. Go to http://geofred. stlouisfed.org. Click on the arrow to get to the map. Click on "Tools" and "Choose Data" and fill in the drop-down menu as follows:

   • Region Type: State

   • Data: Total Gross Domestic Product by State

   • Frequency: Annual

   • Units: Millions of Dollars

   • Date: 2013

   Click on "Edit Legend" and for "Interval Method," choose Equal Interval. Leave the table included under "Edit Legend" exposed. Refer students to the table , as well as the map key, and ask the questions on Part I of Handout 3 as follows. After each question is asked, allow time for students to write their responses in the handout. Use *Handout 3: Assessment—Answer Key* to review student answers.

   • In what units are the data reported on the map key?

   • In what units are the data reported in the table under "Edit Legend?"

   • How many categories of data are there?

   • What is the interval for each category?

- Which color represents the highest amount of Gross Domestic Product by state?

- Which color represents the lowest amount of Gross Domestic Product by state?

- Based on this map, does there appear to be much difference from state to state?

- Why is this an example of equal interval categorization?

22. Under "Edit Legend" change the "Interval Method" to Fractile. Refer students to Part II of Handout 3. Read each question and allow time for students to record their answers on the handout. Refer to Handout 3—Answer Key to check student responses.

    Discuss the following:

    - In what units are the data reported on the map key?

    - In what units are the data reported in the table under "Edit Legend?"

    - How many categories of data are there?

    - What is the interval for each category?

    - Which color represents the highest amount of Gross Domestic Product by state?

    - Which color represents the lowest amount of Gross Domestic Product by state?

    - Based on this map, does there appear to be much difference from state to state?

    - Why is this an example of fractile interval categorization?

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Handout 1: Age Cards

19 years	20 years	21 years	22 years
16 years	17 years	22 years	23 years
30 years	31 years	34 years	42 years
43 years	44 years	45 years	46 years
50 years	51 years	52 years	62 years
49 years	37 years	48 years	67 years

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Handout 2: Equal vs. Fractile

Table A		Table B	
Category/Interval	Number of Cards/ Data Points	Category/Interval	Number of Cards/ Data Points
15-19 years		15-20 years	
20-24 years		21-29 years	
25-29 years		30-39 years	
30-34 years		40-45 years	
35-39 years		46-50 years	
40-44 years		51-69 years	
45-49 years			
50-54 years			
55-59 years			
60-64 years			
65-69 years			

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Handout 2: Equal vs. Fractile—Answer Key

Table A		Table B	
Category/Interval	**Number of Cards/ Data Points**	**Category/Interval**	**Number of Cards/ Data Points**
15-19 years	*16, 17, 19*	15-20 years	*16, 17, 19, 20*
20-24 years	*20, 21, 22, 22, 23*	21-29 years	*21, 22, 22, 23*
25-29 years		30-39 years	*30, 31, 34, 37*
30-34 years	*30, 31, 34*	40-45 years	*42, 43, 44, 45*
35-39 years	*37*	46-50 years	*46, 48, 49, 50*
40-44 years	*42, 43, 44*	51-69 years	*51, 52, 62, 67*
45-49 years	*45, 46, 48, 49*		
50-54 years	*50, 51, 52*		
55-59 years			
60-64 years	*62*		
65-69 years	*67*		

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Handout 3: Assessment

Part I:

Directions: Refer to the GeoFRED map being displayed in class to answer each question.

1. In what units are the data reported on the map key?

2. In what units are the data reported in the table under "Edit Legend?"

3. How many categories of data are there?

4. What is the interval for each category?

5. Which color represents the highest amount of Gross Domestic Product by state?

6. Which color represents the lowest amount of Gross Domestic Product?

7. Based on this map, does there appear to be much difference from state to state?

8. Why is this an example of equal interval categorization?

Handout 3: Assessment—cont.

Part II:

Directions: Refer to the GeoFRED map being displayed in class to answer each question below.

9. In what units are the data reported on the map key?

10. In what units are the data reported in the table under "Edit Legend?"

11. How many categories of data are there?

12. What is the interval for each category?

13. Which color represents the highest amount of Gross Domestic Product by state?

14. Which color represents the lowest amount of Gross Domestic Product by state?

15. Based on this map, does there appear to be much difference from state to state?

16. Why is this an example of fractile interval categorization?

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Handout 3: Assessment—Answer Key

Part I:

Directions: Refer to the GeoFRED map being displayed in class to answer each question.

1. In what units are the data reported on the map key? *millions of dollars*

2. In what units are the data reported in the table under "Edit Legend?" *dollars*

3. How many categories of data are there? *five*

4. What is the interval for each category? *$434,633*

5. Which color represents the highest amount of Gross Domestic Product by state? *the darkest color*

6. Which color represents the lowest amount of Gross Domestic Product? *the lightest color*

7. Based on this map, does there appear to be much difference from state to state? *No, most states fall into the lowest category.*

8. Why is this an example of equal interval categorization? *The range for each category is equal, and the number of data points (states) in each category is different.*

Handout 3: Assessment—Answer Key—cont.

Part II:

Directions: Refer to the GeoFRED map being displayed in class to answer each question below.

9.   In what units are the data reported on the map key? *millions of dollars*

10.   In what units are the data reported in the table under "Edit Legend?" *dollars*

11.   How many categories of data are there? *five*

12.   What is the interval for each category? *category 1: $38,339; category 2: $76,214; category 3: $138,424; category 4: $172,046; and category 5: $1,748,146*

13.   Which color represents the highest amount of Gross Domestic Product by state? *the darkest color*

14.   Which color represents the lowest amount of Gross Domestic Product by state? *the lightest color*

15.   Based on this map, does there appear to be much difference from state to state? *Yes, there is much more variance in the colors of the map. There are more states displayed in the darker colors and fewer states displayed in the lightest color.*

16.   Why is this an example of fractile interval categorization? *The range for each category is different, and the number of data points [states] in each category is equal.*