Appendix: The relation between stock prices, dividend growth and stock returns

This appendix shows a well-known relation between stock prices, dividend growth and stock returns under the assumptions that future dividend growth and expected stock returns are constant.

The following is the accounting definition of a stock return. From period t to $t+1$, the gross return to a stock (or stock index) is the price (P) of the stock plus the dividend (D), divided by the purchase price.

$$1 + R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

We can rearrange this definition to get an expression for stock prices in terms of the price next period, the dividend and the return. Note that there are no expectations in the term below because we are still using the definition of return.

$$P_t = \frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}}$$

This definition must hold for prices for all periods (i.e., t, $t+1$, $t+2$, …).

$$P_t = \frac{P_{t+1}+D_{t+1}}{1+R_{t+1}} \qquad P_{t+1} = \frac{P_{t+2}+D_{t+2}}{1+R_{t+2}} \qquad P_{t+2} = \frac{P_{t+3}+D_{t+3}}{1+R_{t+3}}$$

Note that if we substitute the definition of future prices (P_{t+1}) into the expression for prices at t, we get the first line below. To get the second line, substitute the expression for P_{t+1} in terms of future prices, dividends and returns, i.e., $\left(\frac{P_{t+2}+D_{t+2}}{1+R_{t+2}}\right)$. To get the third line, substitute the expression for P_{t+2} in terms of future prices, dividends and returns, i.e., $\left(\frac{P_{t+3}+D_{t+3}}{1+R_{t+3}}\right)$.

$$P_t = \frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}} = \frac{\frac{(P_{t+2} + D_{t+2})}{(1 + R_{t+2})} + D_{t+1}}{1 + R_{t+1}}$$

$$= \frac{P_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})} + \frac{D_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})} + \frac{D_{t+1}}{1 + R_{t+1}}$$

$$= \frac{P_{t+3}}{(1 + R_{t+1})(1 + R_{t+2})(1 + R_{t+3})} + \frac{D_{t+3}}{(1 + R_{t+1})(1 + R_{t+2})(1 + R_{t+3})} + \frac{D_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})} + \frac{D_{t+1}}{1 + R_{t+1}}$$

If we assume that all future returns are expected to be equal and we recursively substitute in for future prices (P_{t+3}, P_{t+4}, P_{t+5}, ... P_{t+K-1}), then we get an expression for P_t in terms of future expected dividends, returns and the price in *K* periods.

$$P_t = E_t\left[\sum_{i=1}^{K} \left(\frac{1}{1+R}\right)^i D_{t+i}\right] + E_t\left[\left(\frac{1}{1+R}\right)^K P_{t+K}\right]$$

If we can assume that the future price doesn't go to infinity too quickly, that is,

$$\lim_{K \to \infty} E_t \left[\left(\frac{1}{1+R} \right)^K P_{t+K} \right] = 0$$

and that dividend growth (g) and average stock returns (R) are constant, then

$$P_t = E_t \left[\sum_{i=1}^{\infty} \left(\frac{1}{1+R} \right)^i D_{t+i} \right] = E_t \left[\sum_{i=1}^{\infty} \left(\frac{1+g}{1+R} \right)^i D_t \right] = \frac{(1+g)D_t}{R-g}$$

where the third equality comes from the following algebra:

$$\sum_{i=1}^{\infty} \left(\frac{1+g}{1+R} \right)^i = \sum_{i=0}^{\infty} \left(\frac{1+g}{1+R} \right)^i - 1 = \frac{1}{1 - \left(\frac{1+g}{1+R} \right)} - 1$$

$$\frac{1}{1 - \left(\frac{1+g}{1+R} \right)} - 1 = \frac{1}{1 - \left(\frac{1+g}{1+R} \right)} - \frac{1 - \left(\frac{1+g}{1+R} \right)}{1 - \left(\frac{1+g}{1+R} \right)} = \frac{\left(\frac{1+g}{1+R} \right)}{1 - \left(\frac{1+g}{1+R} \right)}$$

$$= \frac{1+g}{(1+R) - (1+g)} = \frac{(1+g)}{R-g}$$

where g is the growth rate of dividends and R is the expected return on stocks. The second line uses the well-known formula for the sum of an infinite geometric series: $\sum_{i=0}^{\infty} ab^i = \frac{a}{1-b}$, where $|b| < 1$. Therefore, the above formula must assume that expected stock returns are greater than expected dividend growth.

So, the result is that—under the assumptions—current stock prices are an increasing function of net dividend growth (g) and a decreasing function of future returns.

$$P_t = \frac{(1+g)D_t}{R-g}$$

We could dispense with constant dividend growth and make the expression more complicated, but higher prices today would still be associated with a higher growth rate of dividends or lower average stock returns in the future. Note that this is a dynamic accounting identity from the definition of return.