

THE OPTIMAL INFLATION TARGET IN AN ECONOMY WITH LIMITED ENFORCEMENT

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Views expressed are those of the author and do not necessarily reflect official positions of the FOMC or the Federal Reserve System.



COMFORT ZONES

- Typical FOMC member statements: 1 to 2 percent inflation would be ideal.
 - "Small positive inflation is the best inflation target."
- Associated with short-term nominal interest rates near 5 percent.
- Economic theory: nominal interest rates should be zero.
- Why the difference?



THEORY AND REALITY: WHY THE DIFFERENCE?

- Japan.
- Summers (1991): Zero bound on nominal interest rates interferes with stabilization policy.
 - Adam and Billi (2006).
- Smith (2002). Deflation causes disintermediation, harming the operation of credit markets.
- We study a version of the latter:
 - A small amount of inflation "deepens financial markets" in a way we make precise.



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- A continuum of infinitely-lived households.



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- Two assets: currency and consumption loans.



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- Default is punished by perpetual exclusion from the loan market but still permits defaulting households to hold nonnegative currency.
- Inflation will make default less attractive, loosening participation constraints and strengthening the credit market.



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- If the planner puts a relative weight greater than zero but less than the population weight on credit households, the optimum inflation rate is positive.
- Conclude: Independent central banks will set low positive inflation targets in economies that possess highly developed financial markets.



Recent related literature

- Aiyagari and Williamson (2000 *JET*).
 - Related environment, random endowments, outside option is Bewley, emphasis on financial intermediation, positive inflation deters default, computational.
- Berentsen, Camera and Waller (2007 *IET*).
 - Related environment, Lagos-Wright, emphasis on financial intermediation, positive inflation deters default.
- Ragot (2007). OLG framework.
- Andolfatto (2007).
- Antinolfi, Azariadis, and Bullard (2007).
 - Money is the only asset, emphasis on dynamics and equilibrium selection.
- Sanchez, Williamson, and Wright (2007). Blackboard version.



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- Continuum of infinitely-lived households.
- λ "Kehoe-Levine" or *credit* type agents.
- 1λ "Townsend-Bewley" or *cash* type agents.
- All types have identical preferences

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \tag{1}$$

with discount factor $0 < \beta < 1$, u(c) standard, and i = 0, 1, 2, 3.



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- Cash households are divided into two sub-types, 2 and 3, with mass $1 - \lambda/2$ each.
- Endowments, interpreted as income shares, are periodic:

$$\left(\omega_t^0, \omega_t^1\right) = \left(\omega_t^2, \omega_t^3\right) = \begin{cases} (1+\alpha, 1-\alpha) & \text{if } t = 0, 2, \dots \\ (1-\alpha, 1+\alpha) & \text{if } t = 1, 3, \dots \end{cases}$$
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- Aggregate income is constant.



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- Credit agents who default are forever excluded from the loan market and must instead use currency.
- Future inflation rates will impact the payoff to default.



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- ... and directly selects consumption vectors for credit agents who
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- The inflation target in this economy is similar to an optimal tax subject to an incentive constraint as understood by Mirrlees (1971).



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- ... and confer two benefits on credit agents:
 - a transfer of resources from the cash sector, and
 - a reduction in the default payoff which brings about higher debt limits.



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- Because inflation is a distortionary tax, we define a *modified* planning problem.



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subject to

$$x_H \le 1 + \alpha, \tag{4}$$

$$x_H + \pi x_L = 1 + \alpha + \pi (1 - \alpha), \qquad (5)$$

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• (1) nonnegative currency; (2) money balances are used up to smooth consumption in low income dates; (3) agents can renounce money and consume endowments.



Let $x_H(\pi)$ and $x_L(\pi)$ solve the previous problem. Given π , the planner now chooses consumption values $(c_H, c_L) \ge 0$ for credit households to maximize the equal-treatment welfare function

$$\frac{1}{1-\beta^2} \left[u \left(c_H \right) + u \left(c_L \right) \right] \tag{7}$$

subject to the resource constraint

$$\lambda (c_H + c_L) + (1 - \lambda) [x_H (\pi) + x_L (\pi)] = 2,$$
 (8)

and the participation constraint

$$u\left(c_{H}\right)+\beta u\left(c_{L}\right)\geq u\left[x_{H}\left(\pi\right)\right]+\beta u\left[x_{L}\left(\pi\right)\right].\tag{9}$$



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 Equal treatment of high income and low income households means that the discounted utilities are weighted equally. This gives the welfare function above.



• If $c_H(\pi)$ and $c_L(\pi)$ solve the previous problem for a given $\pi > 0$, the planner selects the stationary inflation factor π to maximize the social welfare function

$$W(\pi, \delta) = \delta \{ u [c_H(\pi)] + u [c_L(\pi)] \} + (1 - \delta) \{ u [x_H(\pi)] + u [x_L(\pi)] \}.$$
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- A strictly utilitarian SWF would have equal weights for all, that is, $\delta = \lambda$.



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- This gives a *first best* solution with $\pi = \beta$, $R^N = 1$, and smooth consumption for both groups of agents.



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- Suppose $\delta = 1$, no welfare weight on the cash community. Then the planner chooses maximal seigniorage $\tilde{\pi}$ and smooths the consumption of the credit community completely.
- Suppose $\delta = 0$, no welfare weight on the credit community. Then the planner will push the inflation factor as close to zero as possible.



MORE ON SECOND BEST

• The second best trades off these two extreme cases.

THEOREM

The second best optimum allocation under a utilitarian social welfare function satisfies $(c_H, c_L, x_H, x_L) = (c^{**}, c^{**}, x_H (\pi^{**}), x_L (\pi^{**}))$. It is supported by a "modified Friedman rule" for some inflation factor $\pi^{**} \in (\beta, 1)$, and a nominal yield such that $R^N = \pi^{**}/\beta > 1$.

PROOF.

See the appendix.



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 - Perhaps more controversial?
 - "The market for unsecured credit can be made to work perfectly."



LEMMA

Define $W_{\pi}(\pi, \delta) = \partial W/\partial \pi$. Then (a) $W_{\pi}(\pi, \delta) < 0$ $\forall (\pi, \delta) \in [\bar{\pi}, \tilde{\pi}] \times [0, \lambda]$, and (b) $\lim_{\pi \to 1} \mathcal{W}_{\pi}(\pi, \delta) = +\infty$ when π converges from above.

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- Part (*b*) intuition. A small increase in the inflation tax creates a credit market where one would otherwise not exist.



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 $W(\pi, \delta)$ is not defined for $\pi < 1$. It is decreasing in π for $\pi \in (\tilde{\pi}, 1/\bar{R})$ and constant for $\pi \ge 1/\bar{R}$.

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- Deflation violates the participation constraint for high income credit households.
- The outcome of any deflation is that money has a higher payoff than credit.



THEOREM 4

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Suppose assumptions **A1**, **A2**, and **A3** hold, and $0 < \delta \le \lambda$. Then the optimum inflation factor is $\pi^*(\delta) > 1$ and the associated nominal interest yield, $R^N \in (\pi^*(\delta), \pi^*(\delta)/\beta)$, is even higher.



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- Main conclusion: independent central banks will set low positive inflation targets in economies that possess highly developed financial markets.
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- Sounds like "comfort zones" articulated by central bankers.