Closing Small and "Sufficiently" Large Open Economies with Different Asset Structures

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Abstract

There are two important dimensions that matter when we write down a model economy of a country that is open to international financial markets. The first one is its size, and the second one is its asset market structure. Small open economies are price takers so the analysis happens in partial equilibrium, while countries that are "sufficiently" large can affect international prices and the analysis happens in general equilibrium. The second important dimension is the asset market structure. If markets are complete there is full risk sharing, while if markets are incomplete there is not. In this paper I explore how these two dimensions—size and market structure—affect the conditions for the existence and the uniqueness of the equilibrium in economic models, and discuss how to achieve those conditions when they are not present. I finish by discussing how these implications change when a tax on the return to net foreign assets is introduced, generating endogenous incomplete markets.

JEL codes: F41, E44, D52, D53, C62.

Federal Reserve Bank of St. Louis *Review*, Fourth Quarter 2024, Vol. 106, No. 13, pp. 1-8.
https://doi.org/10.20955/r.2024.13

1. INTRODUCTION

What is the difference between a small open economy and a "sufficiently" large open economy? If a country is small, demand and supply decisions in that economy have no bearing on prices for the rest of the world or any other country that they interact with (as long as that country is larger than they are). In other words, small open economies are *price takers*, and the analysis undertaken is in partial equilibrium. However, when we think about a country that is "sufficiently" large to affect world prices, things are different. In these cases, general equilibrium effects are important. Their demand and supply decisions affect the rest of the world, including any other country they interact with. Furthermore, the size of a country and its ability to repay its debts also has a bearing on the degree to which that country can share risk. These differences have implications for the existence and uniqueness of the equilibrium in a model economy.

To illustrate these differences from a theoretical standpoint, in this article I explore and analyze the conditions for the equilibrium in five different scenarios. I start with the small open economy setting with incomplete markets, followed by the small open economy setting with complete markets. I then move to the cases of sufficiently large economies with both incomplete and complete markets; I finish with the case of a sufficiently

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large economy with complete markets and a tax on the return to net foreign assets, which effectively generates endogenous incomplete markets.

I show how small open economy models with incomplete markets have a non-stationarity and discuss different alternatives already present in the literature to resolve this issue. Small open economy models with complete markets do not have this problem because consumption can be pinned down as a fraction of the rest of the world or another country's consumption. Sufficiently large countries, however, have a non-stationarity that cannot be resolved by having complete markets. This is because the consumption of the other country (or of the rest of the world) cannot be taken as exogenous and consumption cannot be pinned down relative to the other country's. In both the complete markets and the incomplete markets cases, only the growth rate of consumption is determined and not the level, implying that the equilibrium is dependent on initial conditions. Finally, I show that in the case of a sufficiently large economy with complete markets and a tax on the return to net foreign assets, initial conditions are not enough. Not only should the tax be zero in the long run, but one of the methods to restore stationarity in a small open economy model with incomplete markets should be used as well.

The article proceeds as follows: Section 2 discusses how to solve the standard small open economy model with complete and incomplete asset markets. Section 3 shows how to solve a standard "sufficiently" large open economy model with different types of asset structures. Section 4 solves the case with complete markets and taxes in a "sufficiently" large open economy. Section 5 concludes.

2. SOLVING THE STANDARD SMALL OPEN ECONOMY MODEL

In this section, I explore the case when a country is small—it does not affect world prices—and compare the case when asset markets are incomplete with the case when asset markets are complete.

2.1 Incomplete Asset Markets

Consider a small open economy populated by a large number of identical households with preferences described by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \frac{\varphi}{1+\gamma} h_t^{1+\gamma} \right\}.$$

The problem of a household is to choose consumption c_t, hours worked h_t, purchases of capital to be rented out next period k_{t+1}, and a risk-free bond b_{t+1} whose price q_{t+1} is taken as given and determined abroad, subject to a flow budget constraint:

$$\begin{aligned} c_t + k_{t+1} + q_{t+1} b_{t+1} \quad \leq \quad & y_t + b_t \\ & + (1-\delta) k_t, \end{aligned}$$

with initial capital k_0 and bonds b_0 given, and the discount factor β being equal to the price of the bond $q = \frac{1}{1+r}$, where r is the risk free interest rate (this assumption holds throughout). Output is produced using capital and hours worked $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$, where A_t is an exogenous stochastic productivity shock.

After deriving the first-order conditions for the household, we can rearrange them to find the optimal condition governing the labor supply,

$$(1-\alpha) A_t \left(\frac{k_t}{h_t}\right)^{\alpha} = \varphi c_t h_t^{\gamma}, \tag{1}$$

the Euler equation for domestic capital,

$$1 = E_t \left[\beta \frac{c_t}{c_{t+1}} \left(\alpha A_{t+1} \left(\frac{h_{t+1}}{k_{t+1}} \right)^{1-\alpha} - 1 + \delta \right) \right], \tag{2}$$

and the Euler equation for bonds,

$$E_t \left[q_{t+1} \frac{c_{t+1}}{c_t} \right] = \beta. \tag{3}$$

If we were to find the steady state of this model (assuming it exists), the system of equations that solves for c, h, k, and b would be given by:

$$(1-\alpha) A \left(\frac{k}{h}\right)^{\alpha} = \varphi c h^{\gamma},$$

$$\frac{1}{\beta} = \left(\alpha A \left(\frac{h}{k} \right)^{1-\alpha} - 1 + \delta \right),$$

$$c + (q-1)\, b \leq Ak^{\alpha} h^{1-\alpha} - \delta k_t,$$

$$\beta = q. \tag{4}$$

However, note that because q is exogenously given, because it was assumed to be equal to r (as well as A), the last equation provides no information; so, we effectively have a system of three equations to solve for four unknowns. Here's an alternative way of thinking about this: Because the rate of return on the risk-free bond is determined abroad and the small open economy is more impatient than the rest of the world (or foreign lenders), the steady state of the model depends on the country's initial net foreign asset position, such that temporary shocks have long-run effects on the state of the economy because the equilibrium dynamics have a random walk component. See Equation (3).

We can resolve this problem by adding a number of modifications to the standard model, such that stationarity is regained. Schmitt-Grohé and Uribe, 2003 discuss four different alternatives to close this small open economy model and their quantitative implications. All solutions rely on adding a functional form to the model, such that Equation (4) is no longer redundant.

The first option is to assume that the discount factor β is endogenous (see Uzawa, 1968, Obstfeld, 1990, Mendoza, 1991, Schmitt–Grohé, 1998, or Uribe, 1997 and decreasing in individual consumption. Agents are more impatient the more they consume. With this assumption, Equation (4) is no longer redundant because it transforms into $\beta(c)(1+r) = 1$, which pins down the steady-state level of consumption solely as a function of r and the parameters defining the function $\beta(.)$. Alternatively, one can assume that the discount factor β is endogenous and decreasing in aggregate consumption, in which case the effect on the Euler Equation (4) is the same; but agents do not internalize the fact that their decisions affect aggregate consumption. Kim and Kose, 2003 compare the business-cycle implications of these models with those of a model with a constant discount factor. They find that both types of models render similar results in terms of the co-movements of macroeconomic aggregates.

Another option is to assume that the interest rate premium is affected by the level of debt. See Senhadji, 1994, Mendoza and Uribe, 2000, or Schmitt-Grohé and Uribe, 2001. In this case, the steady-state Euler equation implies that $\beta[1+r+p(d)] = 1$, where $p(d)$ is the premium over the world interest rate paid by domestic agents and d is the stock of foreign debt. Hence, in this case, the steady state of the net foreign asset position is a function of r and the parameters that define the premium function $p(.)$.

Finally, one can introduce portfolio adjustment costs (see Neumeyer and Perri, 2005), meaning that the cost of increasing asset holdings by one unit is greater than one. In this case, the Euler Equation (4) becomes $1 + \varphi(d) = \beta(1+r)$, where $\varphi(.)$ is the portfolio adjustment cost; and in steady state the level of foreign debt depends only on parameters of the model.

In summary, small open economy models with incomplete markets, where the small open economy is more impatient than its counterpart, have a non-stationarity that has to be resolved by closing the model to induce such stationarity. See Schmitt-Grohé and Uribe, 2003 for a quantitative assessment of the different alternatives discussed above.

2.2 Complete Asset Markets

When markets are incomplete as in Section 2.1, agents have access to a single financial asset that pays a risk-free real rate of return. When markets are complete, agents have access to a complete array of state-contingent claims. As we will see, when an economy is small relative to the other countries, the complete market assumption per se induces stationarity in the equilibrium dynamics, and we do not have to close the model as was discussed in the previous section.

To see the difference between the incomplete markets case and the complete markets case, lets look at how to solve the latter. Preferences and technology are the same as in the previous section, such that the utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \frac{\varphi}{1+\gamma} h_t^{1+\gamma} \right\};$$

but now, the period-by-period budget constraint of the household is given by

$$c_t + k_{t+1} + E_t\left[q_{t+1}b_{t+1}\right] \leq y_t + b_t + (1-\delta)\,k_{jt}, \tag{5}$$

where b_{t+1} denotes a random variable indicating the number of assets purchased in period t to be delivered in each state of period $t+1$. The variable q_{t+1} is a risk-adjusted world price multiplied by the probability of the state occurring, which allows us to write the expected value of the risk-adjusted expenditures on bonds on the left-hand side of the flow budget constraint. Again, output is produced using capital and hours worked$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$, where A_t is an exogenous stochastic productivity shock.

After deriving the first-order conditions for the household, we can rearrange them to find the optimal condition governing the labor supply,

$$(1-\alpha)\,A_t\left(\frac{k_t}{h_t}\right)^{\alpha} = \varphi c_t h_t^{\gamma}, \tag{6}$$

the Euler equation for domestic capital,

$$1 = E_t\left[\beta\frac{c_t}{c_{t+1}}\left(\alpha A_{t+1}\left(\frac{h_{t+1}}{k_{t+1}}\right)^{1-\alpha} - 1 + \delta\right)\right], \tag{7}$$

and the Euler equation for bonds,

$$q_{t+1}\frac{c_{t+1}}{c_t} = \beta, \tag{8}$$

which holds for every state of the world in period $t+1$, while in the case with incomplete markets, Equation (3) holds only in expectations.

In the rest of the world, agents have access to the same portfolio of financial assets as in the domestic economy, so every other country in the world has a first-order condition with respect to bonds that is analogous to Equation (8)

$$q_{t+1}\frac{c^*_{t+1}}{c^*_t} = \beta,$$

where c^*_t denotes foreign consumption in period t. Because the price of the bond q_{t+1} and the discount factor β are common to all the countries, then

$$\frac{c_{t+1}}{c_t} = \frac{c^*_{t+1}}{c^*_t},$$

at all dates under all contingencies. This implies that the domestic marginal utility of consumption is proportional to its foreign counterpart

$$c_t = \theta c^*_t,$$

where θ is a constant parameter determining differences in wealth across countries. Because the domestic economy is small and cannot affect world allocations, c^*_t is an exogenous variable that can be assumed constant and equal to c^* because we only care about the effects of domestic productivity shocks such that

$$c_t = \theta c^*. \tag{9}$$

This means that the system of equations that determines a stationary equilibrium is given by Equation (9), plus Equations (7), (6), and (5) , and there is no need to close the model.

3. SOLVING THE STANDARD SUFFICIENTLY LARGE OPEN ECONOMY MODEL

In this section, we explore the difference that having complete versus incomplete asset markets makes for the equilibrium of a sufficiently large open economy.

Consider a world economy composed of two large countries indexed by j. Time evolves discretely and is indexed by $t = 0, 1, ...$ The decisions of each country are made by a representative agent with preferences over consumption c_{jt} and hours worked h_{jt} ordered by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left(c_{jt}\right) - \frac{\varphi}{1+\gamma} h_{jt}^{1+\gamma} \right\}.$$

The parameters governing preferences—the discount factor β, the preference for leisure φ, and the Frisch elasticity of labor supply 1/γ—are assumed common across countries.

The difference between there being market incompleteness and complete markets, as stated in Section 2, relies on the budget constraint. Under incomplete markets, the problem of the representative agent of country j is to choose a state-contingent stream of consumption levels c_{jt}, hours worked h_{jt}, purchases of capital to be rented out next period k_{jt+1}, and a risk-free bond b_{t+1} that pays one unit of consumption in every state of the world and whose price q_{t+1} is determined abroad, subject to a flow budget constraint:

$$\begin{aligned} c_{jt} + k_{jt+1} + q_{t+1} b_{jt+1} \quad \leq \quad & y_{jt} + b_{jt} \\ & + (1-\delta)\, k_{jt}, \end{aligned}$$

with initial capital k_{j0} and bonds b_{j0} given. Output is produced using capital and hours worked $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$, where A_t is an exogenous stochastic productivity shock.

In the complete markets case, the problem of the representative agent of country j is to choose a state-contingent stream of consumption levels c_{jt}, hours worked h_{jt}, purchases of capital to be rented out next period k_{jt+1}, and a portfolio of state-contingent international bond holdings b_{jt+1} subject to a sequence of flow budget constraints for each state and date:

$$\begin{aligned} c_{jt} + k_{jt+1} + E_t\left[q_{t+1} b_{jt+1}\right] \quad \leq \quad & y_{jt} + b_{jt} \\ & + (1-\delta)\, k_{jt}, \end{aligned}$$

with initial capital K_{j0} and bonds B_{j0} given. The same assumptions about production technology also hold. In this complete markets environment, the prices of state-contingent international bonds at time t that pay off in one state at $t + 1$ are composed of a risk-adjusted world price q_{t+1} multiplied by the probability of the state occurring, which allows us to write the expected value of the risk-adjusted expenditures on bonds on the left-hand side of the flow budget constraint.

The difference between these two market structures relies on the Euler equation for international assets. When markets are incomplete, this is given by

$$E_t\left[q_{t+1} \frac{c_{jt+1}}{c_{jt}}\right] = \beta, \ \forall j, \tag{10}$$

and when markets are complete, the Euler equation for state-contingent international assets is given by

$$\frac{c_{jt+1}}{c_{jt}} = \frac{\beta}{q_{t+1}}, \ \forall j, t \ and\ state. \tag{11}$$

Note that this means that, under complete markets, there is an infinite number of Euler equations, as there is one for every state of the world in every period t.

Unlike the small open economy case, when a country is sufficiently large, their demand and supply decisions affect world prices. Hence the situation described in Subsection 2.2, where consumption could be pinned down as a fraction of the rest of the world's consumption, which was taken as exogenous, does not hold. Here, both Equations (10) and (11) imply that the growth rate of consumption has to be the same for country j and country i, although when markets are incomplete this is only true in expectation, while when markets are complete this is exactly true in every state of the world.

Because only relative consumption is pinned down (not consumption levels), the steady state in these two cases is determinate and unique given initial conditions.

4. COMPLETE MARKETS WITH TAXES IN A SUFFICIENTLY LARGE OPEN ECONOMY

What happens when the country is sufficiently large, there are complete asset markets, and there is also a tax on the return to net foreign assets? Initial conditions are not enough. Let's see why.

Consider a world economy composed of two large countries indexed by j. Time evolves discretely and is indexed by $t = 0, 1, ...$ The decisions of each country are made by a representative agent with preferences over consumption c_{jt} and hours worked h_{jt}, ordered by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left(c_{jt}\right) - \frac{\varphi}{1+\gamma} h_{jt}^{1+\gamma} \right\}.$$

The parameters governing preferences—the discount factor β, the preference for leisure φ, and the Frisch elasticity of labor supply $1/\gamma$—are assumed common across countries.

The problem of the representative agent of country j is to choose a state-contingent stream of consumption levels c_{jt}, hours worked h_{jt}, purchases of capital to be rented out next period k_{jt+1}, and a portfolio of state-contingent international bond holdings b_{jt+1} subject to a sequence of flow budget constraints for each state and date:

$$\begin{aligned} c_{jt} + k_{jt+1} + E_t\left[q_{t+1} b_{jt+1}\right] \quad \leq \quad & y_{jt} + \left(1 - \tau_{jt}\right) b_{jt} \\ & + (1-\delta) k_{jt}, \end{aligned}$$

with initial capital K_{j0} and bonds B_{j0} given, output produced using capital and hours worked $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$, and A_t an exogenous stochastic productivity shock. In this complete markets environment, the prices of state-contingent international bonds at time t that pay off in one state at $t+1$ are composed of a risk-adjusted world price q_{t+1} multiplied by the probability of the state occurring, which allows us to write the expected value of the risk-adjusted expenditures on bonds on the left-hand side of the flow budget constraint. Finally, τ_{jt} represents a distortion that is isomorphic to a tax/subsidy on the return to net foreign assets, and it varies over time according to an autoregressive process of order one. See Ohanian, Restrepo-Echavarria, and Wright, 2018.

The Euler equation for state-contingent international assets implies that

$$\frac{c_{jt+1}}{c_{jt}} = \frac{\beta}{q_{t+1}} \left(1 - \tau_{jt+1}\right),$$

and because the bond price (q) and the discount factor (β) are the same for both countries, then

$$\left(\frac{c_{jt+1}}{c_{it+1}}\right)\left(\frac{c_{jt}}{c_{it}}\right) = \frac{1-\tau_{jt+1}}{1-\tau_{it+1}}. \tag{12}$$

Because there is only one Equation (12) for each state and date, we can only identify the tax of country j relative to country i, so from now on we will denote $\frac{1-\tau_{jt+1}}{1-\tau_{it+1}} = 1 - \tau_{t+1}$, such that

$$\left(\frac{c_{jt+1}}{c_{it+1}}\right)\left(\frac{c_{jt}}{c_{it}}\right) = 1 - \tau_{t+1}. \tag{13}$$

This condition implies that independent of initial conditions, there is a long-run trend in relative consumption levels so that the deterministic steady-state distribution of consumption is degenerate; that is, one country's share of consumption must converge to zero. Unlike the case in Section 3, where it was enough to know the initial conditions to solve for the steady-state distribution of consumption, when you add a tax on the return to net foreign assets it is also necessary to assume that there is no tax in the long-run. In other words, the steady state of the tax has to be zero, $\tau_{ss} = 0$.

However, if $\tau_{ss} = 0$ guarantees the existence of a steady state, it does not guarantee a *unique* steady state relative consumption level. Intuitively, the level of τ out of steady state affects the accumulation of international assets, which in turn affects long-run consumption levels. In terms of equation (13), the *growth rate* of relative consumption is a first-order autoregressive process that converges to zero in the deterministic steady state; the long-run *level* of relative consumption depends upon the entire sequence of realizations of the tax on the return to net foreign assets (τ_t).

Analogous issues arise in multi-agent models with heterogeneous rates of time preference (see the conjecture of Ramsey, 1928, the proof of Becker, 1980, and the resolution of Uzawa, 1968), and note that it is a similar situation to what happens in small open economy, incomplete markets models. As mentioned in Section 2.1, in the latter context, a suite of alternative resolutions of this issue has been proposed. See Schmitt-Grohé and Uribe, 2003 for a survey and discussion. In this context, one can choose an available option to close small open economy models while adjusting it to this multi-country setting. Here, we will show how a variant of the portfolio adjustment cost approach is adapted to this general equilibrium, complete markets setting. Specifically, one can assume that the tax on the return to net foreign assets can be decomposed into a pure tax on international investment income τ_t^* and another term Ψ_t, both of which the country takes as given:

$$1 - \tau_t = 1 - \tau_t^* + \Psi_t.$$

Furthermore, it is still the case that τ_t^* follows a first-order autoregressive process with the steady state assumed to be zero,

$$\ln\left(1 - \tau_{t+1}^*\right) = \rho \ln\left(1 - \tau_t^*\right) + \sigma\varepsilon_{t+1}, \tag{14}$$

and the other term takes the form of a portfolio tax that is assumed, in equilibrium, to satisfy

$$\Psi_t = \left(1 - \tau_t^*\right)\left[\left(\frac{c_{jt}}{c_{it}\psi_0}\right)^{-\psi_1} - 1\right]. \tag{15}$$

This ensures that, in the deterministic steady state, relative consumption levels are pinned down by ψ_0, with mean reversion in relative consumption levels controlled by ψ_t as

$$\ln\frac{c_{jt+1}}{c_{it+1}} = \frac{\psi_1}{1+\psi_1}\ln\psi_0 + \frac{1}{1+\psi_1}\ln\frac{c_{jt}}{c_{it}} + \frac{1}{1+\psi_1}\ln\left(1 - \tau_{t+1}^*\right). \tag{16}$$

We refer to this as a portfolio tax because in steady state, relative consumption levels map one-for-one into net foreign asset positions. Under these assumptions on the portfolio tax, there exists a unique non-degenerate deterministic steady state.

5. CONCLUSION

This article discusses the existence and uniqueness of equilibrium in small open economy models versus models of sufficiently large economies with both complete and incomplete markets.

It shows that small open economy models with incomplete markets have a non-stationarity, while small open economy models with complete markets do not have this problem, because consumption can be pinned down as a fraction of the rest of the world or another country's consumption. Section 2 discusses the different alternatives to resolve the non-stationarity when there is market incompleteness.

The article continues to show that in larger economies with complete markets, the inability to treat foreign consumption as exogenous perpetuates the non-stationarity present in small open economies with incomplete markets. As a result, when an economy is sufficiently large—no matter the market structure—only consumption growth rates are determined, such that the equilibrium depends on initial conditions. Additionally, in sufficiently large economies with complete markets and a tax on net foreign asset returns, initial conditions are not enough. Not only should the tax be zero in the long run, but one of the methods to restore stationarity in a small open economy model with incomplete markets should be used as well.

REFERENCES

Becker, Robert A. 1980. On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. *The Quarterly Journal of Economics* 95 (2): 375–382.

Kim, Sunghyun H., and M.A. Kose. 2003. Dynamics of open-economy business-cycle models: role of the discount factor. *Macroeconomic Dynamics* 7 (2): 263–290.

Mendoza, Enrique G. 1991. Real business cycles in a small open economy. *The American Economic Review,* 797–818.

Mendoza, Enrique G., and Martin Uribe. 2000. Devaluation risk and the business-cycle implications of exchange-rate management. In *Carnegie-rochester conference series on public policy,* 53:239–296. 1. Elsevier.

Neumeyer, Pablo A., and Fabrizio Perri. 2005. Business cycles in emerging economies: the role of interest rates. *Journal of monetary Economics* 52 (2): 345–380.

Obstfeld, Maurice. 1990. Intertemporal dependence, impatience, and dynamics. *Journal of Monetary Economics* 26 (1): 45–75.

Ohanian, Lee E., Paulina Restrepo-Echavarria, and Mark LJ Wright. 2018. Bad investments and missed opportunities? postwar capital flows to asia and latin america. *American Economic Review* 108 (12): 3541–3582.

Ramsey, Frank P. 1928. A mathematical theory of saving. *The economic journal* 38 (152): 543–559.

Schmitt–Grohé, Stephanie. 1998. The international transmission of economic fluctuations:: effects of us business cycles on the canadian economy. *Journal of international Economics* 44 (2): 257–287.

Schmitt-Grohé, Stephanie, and Martın Uribe. 2001. Stabilization policy and the costs of dollarization. *Journal of money, credit and banking,* 482–509.

———. 2003. Closing small open economy models. *Journal of International Economics* 61 (1): 163–185.

Senhadji, Abdelhak S. 1994. *Adjustment of a small open economy to external shocks.* University of Pennsylvania.

Uribe, Martın. 1997. Exchange-rate-based inflation stabilization: the initial real effects of credible plans. *Journal of monetary Economics* 39 (2): 197–221.

Uzawa, Hirofumi. 1968. Time preference, the consumption function, and optimum asset holdings. In *Value, capital and growth,* 485–504. The University of Edinburgh Press.