

Turbulent Years for U.S. Banks: 2000-20

Paul W. Wilson

The first 20 years of the twenty-first century have presented U.S. banks with three recessions, long periods of very low interest rates, and increased regulation. The number of commercial banks operating in the United States declined by 51 percent during this period. This article examines the performance of U.S. commercial banks from 2000 through 2020. An overall picture is provided by examining the evolution of assets, deposits, loans, and other financial characteristics over the period. In addition, new estimates of technical inefficiency are provided, offering additional insight into banks' performance during the recent difficult years. (JEL C14, G01, G21)

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1 INTRODUCTION

The first two decades of the twenty-first century have been turbulent for U.S. commercial banks. Banks have confronted recessions in 2001, 2007-09, and 2020, increased regulation, unprecedented periods of low interest rates, and other disruptions. The number of Federal Deposit Insurance Corporation (FDIC) insured commercial banks and savings institutions fell from 10,222 at the end of the fourth quarter of 1999 to 5,002 at the end of the fourth quarter of 2020. During the same period, among the 5,220 banks that disappeared, 571 exited the industry because of failures or assisted mergers,¹ while the creation of new banks slowed. From 2000 through 2007, 1,153 new bank charters were issued, and in 2008 and 2009, 90 and 24 new charters were issued, respectively. But from 2010 through 2020, only 48 new commercial bank charters were issued. The decline in the number of institutions since 2000 continues a long-term reduction in the number of banks operating in the United States since the mid-1980s.

Banks are a critical part of the U.S. economy and, among other roles, serve as financial intermediaries for all types of businesses. Banks in effect arbitrage financial capital by renting funds from depositors and renting funds to borrowers. The rents received on either side depend on prevailing interest rates and in particular on the spread between rates on deposits and loans. This role also

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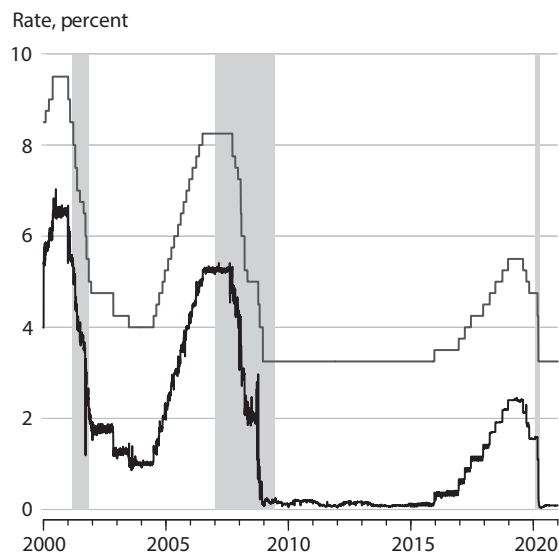
requires that banks manage risk to avoid becoming insolvent. Diamond and Rajan (2001) note that banks perform valuable functions on both sides of their balance sheets. Banks provide liquidity on demand to depositors while at the same time making loans to illiquid borrowers, thereby enhancing the flow of credit in the economy. Despite the unusually low interest rates experienced during 2009-16 and 2020, banks remain an important and substantial part of the U.S. economy. As shown below, financial sector profits accounted for approximately 24.5 percent of corporate profits at the beginning of 2000. The share of corporate profits accruing to the financial sector fluctuated widely during 2000-20, but at the end of 2020 still accounted for approximately 23.2 percent of corporate profits, down only slightly from the beginning of 2020.

This article examines the performance of commercial banks over 2000-20. The next section provides a look at the “big picture” by examining the U.S. banking industry as a whole and how the industry has evolved during the turbulent period from 2000 to 2020. Section 3 presents a model of how individual banks convert inputs (i.e., labor, financial capital, and physical capital) into outputs (i.e., loans, securities, and off-balance-sheet activities) while allowing for risk. The specified model is fully nonparametric, thereby allowing more flexibility and fewer assumptions than more traditional parametric models. Section 4 discusses estimation of the model. The estimation method is almost fully nonparametric; a local assumption on the distribution of efficiency is needed for identification of expected efficiency, but the parameters of the distribution are functional and are estimated locally. The noise term in the model is assumed to have a symmetric density, but no functional form assumptions (even local ones) are needed. The data used for estimation and specification of variables are described in Section 5. Section 6 presents the estimation results, which are discussed in view of the big picture shown in Section 2. Summary and conclusions are given in Section 7.

2 AN OVERALL VIEW OF THE U.S. BANKING INDUSTRY

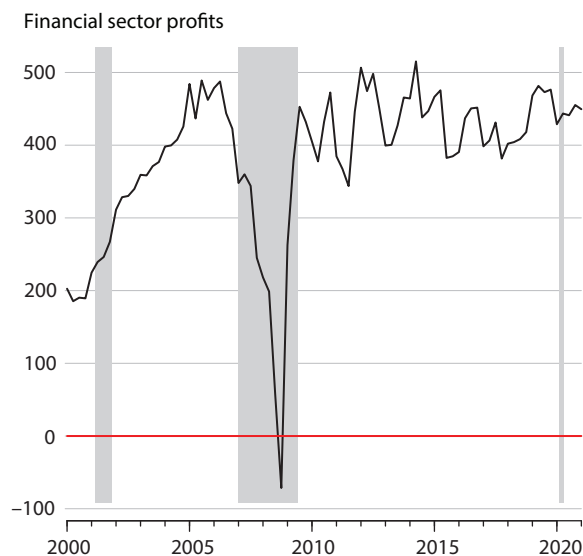
The National Bureau of Economic Research (NBER) dates U.S. recessions starting at the peak and ending at the next trough of a business cycle. The NBER indicates that from 2000 to 2020, recessions occurred from March to December 2001, December 2007 to June 2009, and February to April 2020.² The first of these three recessions was slight and brief, with real gross domestic product (GDP) declining by 0.11 percent from its peak at the end of the third quarter of 2000 to the trough at the end of the second quarter of 2001 and then surpassing the previous peak level in the third quarter of 2001. The second recession—the Great Recession—was more severe. Real GDP declined by 3.84 percent between the end of the third quarter of 2007 and the end of first quarter of 2009 and did not reach the previous peak level until the third quarter of 2010. The third recession saw the largest decline in real GDP—13.12 percent from the end of the third quarter of 2019 to the end of the second quarter of 2020—but by the end of 2020, real GDP had recovered to 99.24 percent of the previous peak level. To the extent that banks serve as financial intermediaries in the economy by smoothing the flow of capital from lenders to borrowers, one might expect that both the Great Recession and the recession of 2020 had significant impacts on banks’ operations.

The federal funds rate is one of the Federal Reserve System’s primary policy levers, and it is well known that the Fed used this tool intensively in recent years. The federal funds rate heavily influences the bank prime loan rate, that is, the interest rate that banks charge their most credit worthy customers. Figure 1 plots both the effective federal funds rate and the prime rate as functions

Figure 1**Prime and Effective Federal Funds Rates**

NOTE: The bank prime loan rate (lighter-colored line) is the rate posted by a majority of top 25 (by assets in domestic offices) insured U.S.-chartered commercial banks. The effective federal funds rate (darker, more variable line) is the interest rate at which depository institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight. The federal funds rate influences the higher prime rate. Gray bars indicate recessions as determined by the NBER.

SOURCE: Board of Governors of the Federal Reserve System (2021a,b).

Figure 2**Financial Sector Profits**

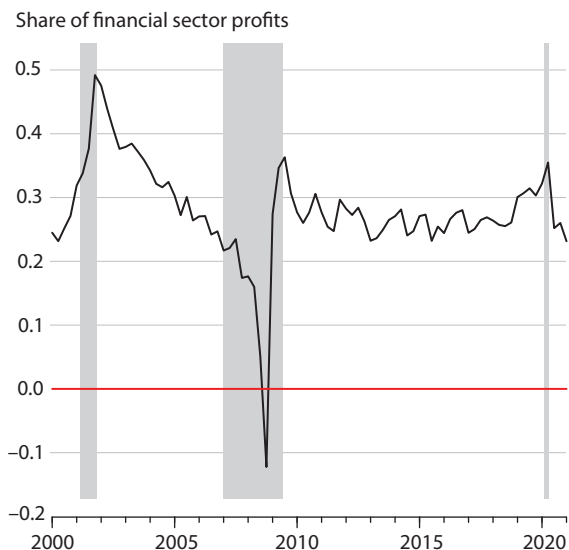
NOTE: Financial sector profits are measured in billions of constant 2012 dollars, seasonally adjusted annual rate. Financial sector profits are computed by subtracting nonfinancial corporate profits before tax from corporate profits before tax. In both cases, profits are not adjusted for inventory valuation nor for capital consumption. Gray bars indicate recessions as determined by the NBER.

SOURCE: U.S. Bureau of Economic Analysis (2021a,d).

of time. The effective federal funds rate fluctuates somewhat in the short run, and Figure 1 shows the effects of Federal Reserve policy on interest rates. The federal funds rate remained below 0.5 percent from October 29, 2008, until December 15, 2016. During much of this period, the rate was below 0.1 percent. The federal funds rate was allowed to rise beginning in 2016, reaching a high of 2.41 percent in July 2019. By March 1, 2020, the federal funds rate stood at 1.58 percent, then plunged to 0.08 percent by the end of March. Recall that in the United States, March 2020 saw the beginning of lockdowns and other restrictions caused by the COVID-19 pandemic. From the beginning of the pandemic until mid-September 2021, the federal funds rate remained at or below 0.1 percent. Figure 1 also illustrates how the prime rate moves in lockstep with the federal funds rate.

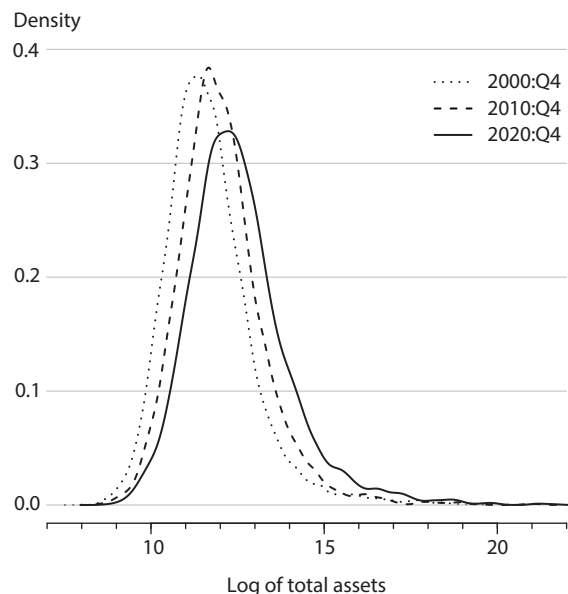
In principle, low interest rates on loans constrain the spread between rates on loans and deposits, and consequently, low interest rates tend to limit banks' ability to earn profits. But as noted in Section 1, financial sector profits rose from 2000 to 2020. Figure 2 plots financial sector profits as a function of time. It shows that financial sector profits increased prior to the Great Recession but then plunged deeply, becoming negative in the third quarter of 2008 and then recovering over the next three quarters. Financial sector profits then oscillated up and down, reaching \$452.8 billion in the second quarter of 2009, ending at \$449.5 billion at the end of 2020.³

A somewhat different picture is provided by Figure 3, which shows the ratio of financial sector profits to total corporate profits. Figure 3 reveals that the financial sector took an increasing share of

Figure 3**Ratio of Financial Sector Profits to Total Corporate Profits**

NOTE: Financial sector profits are computed by subtracting nonfinancial corporate profits before tax from corporate profits before tax. In both cases, profits are not adjusted for inventory valuation nor for capital consumption; data are seasonally adjusted at an annual rate. Gray bars indicate recessions as determined by the NBER.

SOURCE: U.S. Bureau of Economic Analysis (2021a,d).

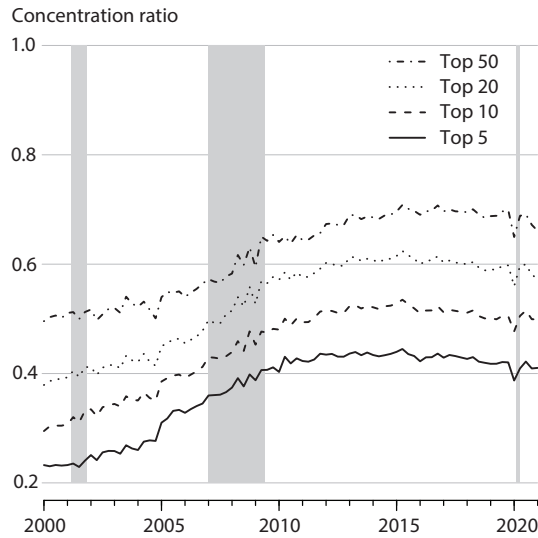
Figure 4**Density of (Log) Total Assets of U.S. Commercial Banks in the Fourth Quarters of 2000, 2010, and 2020**

NOTE: Total assets are measured in billions of constant 2012 U.S. dollars.

total corporate profits up to the third quarter of 2009. But starting in the fourth quarter of 2009, the financial sector's share of corporate profits declined, becoming negative (due to the negative profits mentioned above) in the third quarter of 2008. As a share of total corporate profits, financial sector profits recovered after 2008 but did not reach the levels of 2001-03. Nonetheless, the financial sector remains an important part of the U.S. economy, accounting for a quarter or more of corporate profits.

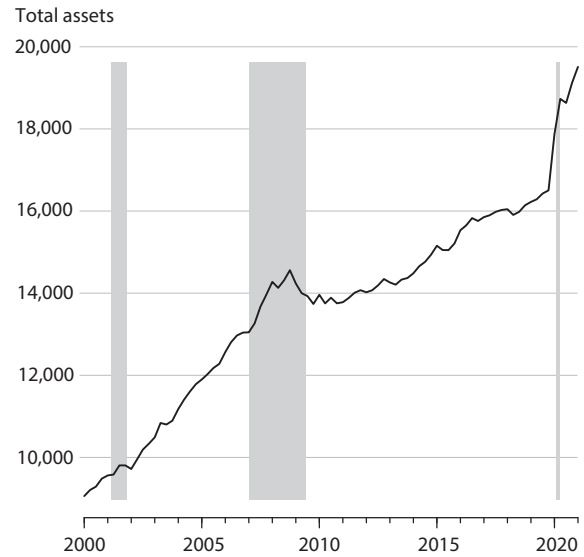
As mentioned in Section 1, there has been a great deal of consolidation among U.S. banks. Wheelock (2011) discusses the effects of the Great Recession on bank consolidation through 2010, and the Bank for International Settlements (2018) discusses bank concentration after the Great Recession. Others, such as Janicki and Prescott (2006) and Goddard, McKillop, and Wilson (2014), examine changes in the size distribution of U.S. banks. It is well known that since the 1980s, banks have become fewer in number and larger in size. Figure 4 shows kernel density estimates of the log of total assets of U.S. banks in the fourth quarters of 2000, 2010, and 2020. Even after taking logarithms, the distribution of total assets remains skewed to the right. Figure 4 illustrates how the density of total assets has shifted rightward over time. In addition, the density for 2020 reaches a maximum that is lower than the maxima in 2000 or 2010, reflecting the fact that the variance of log total assets changed little from 2000 to 2010 (the corresponding variances are 1.643 and 1.625, respectively) but increased by more than 30 percent (to 2.140) in 2020. In addition, the right tail of the density of log total assets is thicker in 2020 than in either 2000 or 2010.

Figure 5
Concentration Ratios in Terms of Total Assets of U.S. Commercial Banks



NOTE: Concentration ratios are computed by dividing the sum of total assets of the largest commercial banks by the sum of total assets of all commercial banks in a given period, where $m \in \{5, 10, 20, 50\}$. Gray bars indicate recessions as determined by the NBER.

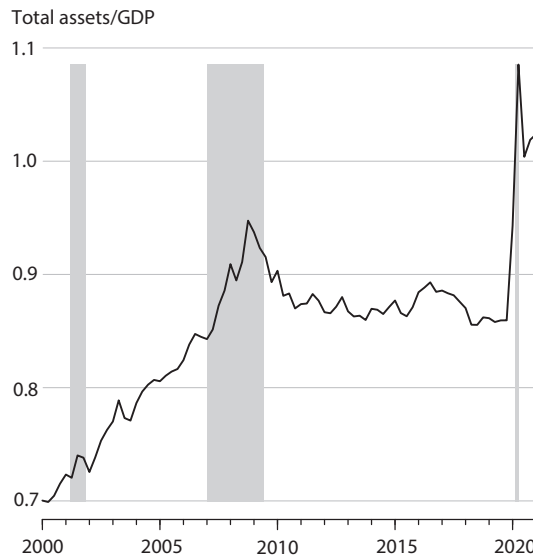
Figure 6
Total Assets of U.S. Commercial Banks



NOTE: Total assets are measured in billions of constant 2012 U.S. dollars, not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021d) and U.S. Bureau of Economic Analysis (2021c).

Figure 7
Ratio of Total Assets of U.S. Commercial Banks to GDP



NOTE: Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021d) and U.S. Bureau of Economic Analysis (2021b).

Figure 5 shows concentration ratios for large U.S. banks as functions of time. For a given point in time, the concentration ratio is computed as the sum of total assets of the m largest banks divided by the sum of total assets of all banks. Figure 5 shows concentration ratios for the top 5, 10, 20, and 50 banks in each quarter. The concentration ratios show an upward trend through 2010 or 2011, but become flat or even decrease slightly from 2011 to 2020. Nonetheless, the five largest banks at the beginning of 2000 controlled 23 percent of bank assets, while the five largest at the end of 2020 controlled 41 percent of bank assets. The largest banks became much larger from 2000 to 2020.

Additional evidence on the increasing size of banks is provided by Figures 6 and 7. Figure 6 shows that total assets of all commercial banks trended upward from 2000 to 2020, especially just before the pandemic-induced recession in 2020. Overall, assets held by commercial banks increased from \$9.2 trillion at the end of the fourth quarter of 1999 to \$19.5 trillion at the end of the fourth quarter of 2020. Of course, GDP also increased over the same period. Figure 7 shows the ratio of total bank assets to GDP as a function of time. The ratio of assets to GDP peaked at 0.9475 in the third quarter of 2008 and did not reach this level again until the first quarter of 2020 before peaking at 1.0853 at the end of the first quarter of 2020.

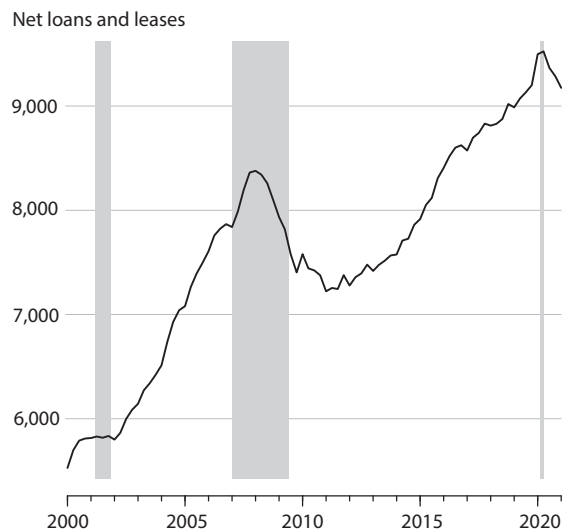
In order to provide some insight into what drove the increase in total bank assets in Figure 6, Figures 8, 9, and 10 show net loans and leases, total securities, and cash assets as functions of time. Figure 8 shows that net loans and leases peaked at \$8.378 trillion in the fourth quarter of 2007, then fell to \$7.223 trillion in the fourth quarter of 2010. Net loans and leases reached the previous peak level in the fourth quarter of 2015 and climbed to \$9.525 trillion in the first quarter of 2020 before falling again.

Figures 9 and 10 show different trajectories for total securities and for cash assets, respectively. Whereas net loans and leases experienced a sharp decline during and after the Great Recession, banks increased their holdings of securities and cash assets during and after the recession. In addition, whereas net loans and leases declined after the first quarter of 2020, total securities held by banks increased from \$3.524 trillion at the end of the third quarter of 2019 to \$4.738 trillion at the end of the fourth quarter of 2020. Cash assets held by banks increased from \$1.476 trillion to \$3.138 trillion over the same period.

Figure 11 shows the composition of total assets. The solid curve indicates the proportion of total assets consisting of net loans and leases, and the dashed curve indicates the part of total assets made up of net loans and leases plus securities; hence, the distance between the solid and dashed curves indicates the proportion of total assets consisting of securities. The dotted line represents net loans and leases plus securities plus cash assets divided by total assets; the distance between the dotted and dashed curves indicates the proportion of total assets consisting of cash assets. The distance between 1.0 and the dotted curve indicates the proportion of other assets, including premises and fixed assets, other real estate owned, investments in unconsolidated subsidiaries and associated companies, and intangible assets. As the figure shows, net loans and leases as a proportion of total assets declined from 61.02 percent at the beginning of 2000 to 47.02 percent at the end of 2020. Net loans and leases plus securities declined from 80.02 percent to 71.31 percent of total assets over the same period, with securities increasing from 19.13 percent to 24.28 percent of total assets. At the beginning of 2020, the sum of net loans and leases securities, and cash assets accounted for 85.06 percent of total assets and at the end of 2020 accounted for 87.39 percent of total assets, implying that other assets as a percentage of total assets also changed little from 2000 to 2020. However, cash

Figure 8

Net Loans and Leases of U.S. Commercial Banks

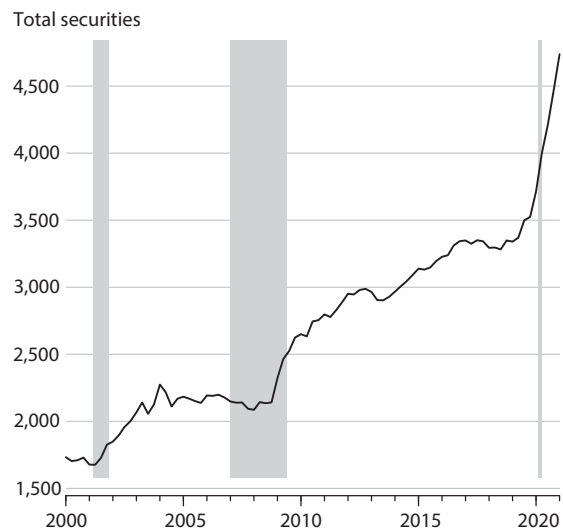


NOTE: Net loans and leases include gross loans and leases less unearned income and reserve for losses, and are measured in billions of constant 2012 U.S. dollars, not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021c) and U.S. Bureau of Economic Analysis (2021c).

Figure 9

Total Securities Held by U.S. Commercial Banks

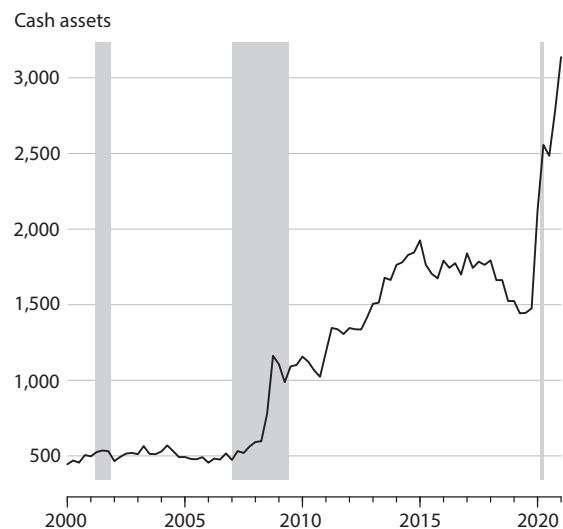


NOTE: Total securities include securities available for sale (fair value), securities held to maturity (amortized cost), U.S. Treasury securities, mortgage-backed securities, state and municipal securities, and equity securities and are measured in billions of constant 2012 U.S. dollars, not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021e) and U.S. Bureau of Economic Analysis (2021c).

Figure 10

Cash Assets from Depository Institutions, U.S. Commercial Banks

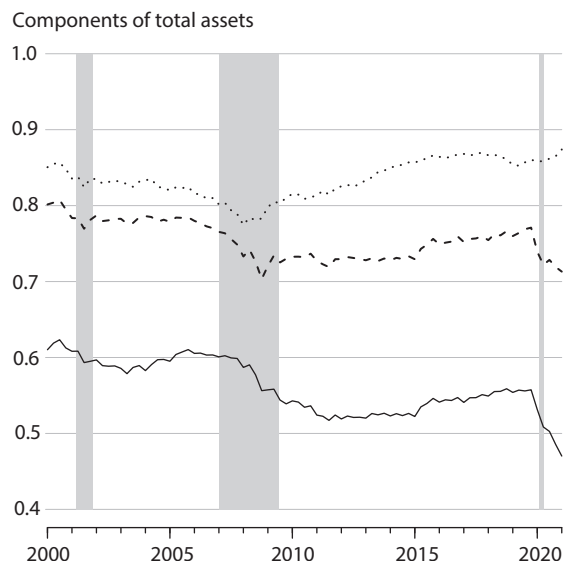


NOTE: Cash assets include cash and due from depository institutions. Cash assets are measured in billions of constant 2012 U.S. dollars, not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021b) and U.S. Bureau of Economic Analysis (2021c).

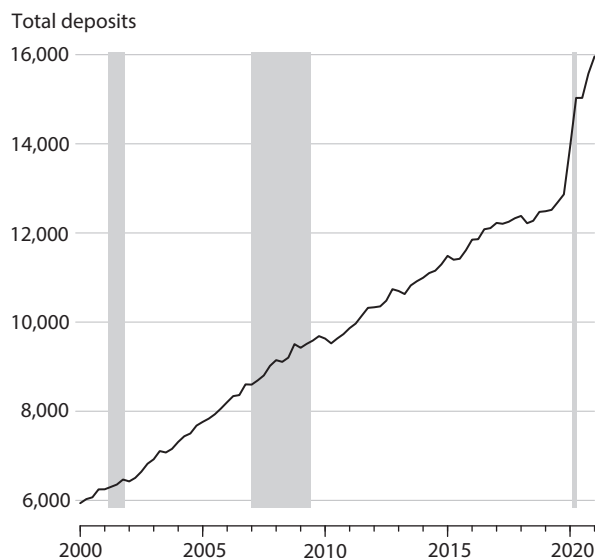
Figure 11

Components of Total Assets of U.S. Commercial Banks



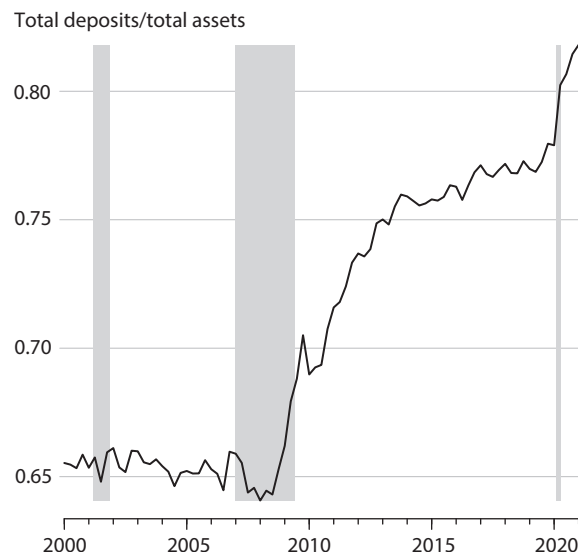
NOTE: The solid curve shows the ratio of net loans and leases to total assets. The dashed curve shows the ratio of net loans and leases plus securities to total assets. The dotted curve shows the ratio of net loans and leases plus securities plus cash assets to total assets. Data are not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021b,c,d,e).

Figure 12**Total Deposits of U.S. Commercial Banks**

NOTE: Deposits are measured in billions of constant 2012 dollars, not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021f).

Figure 13**Ratio of Total Deposits to Total Assets, U.S. Commercial Banks**

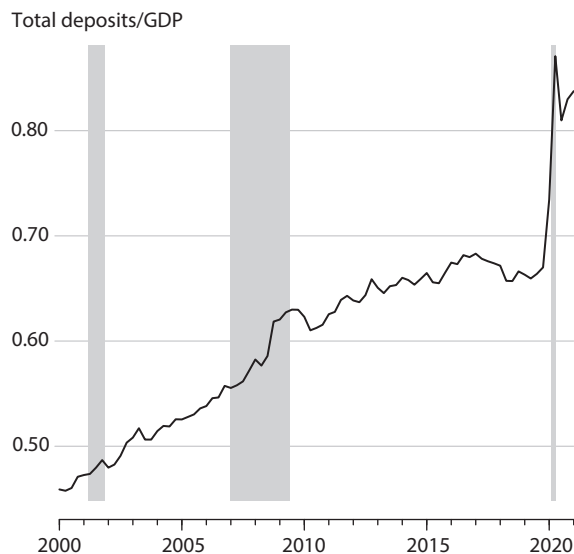
NOTE: Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021f).

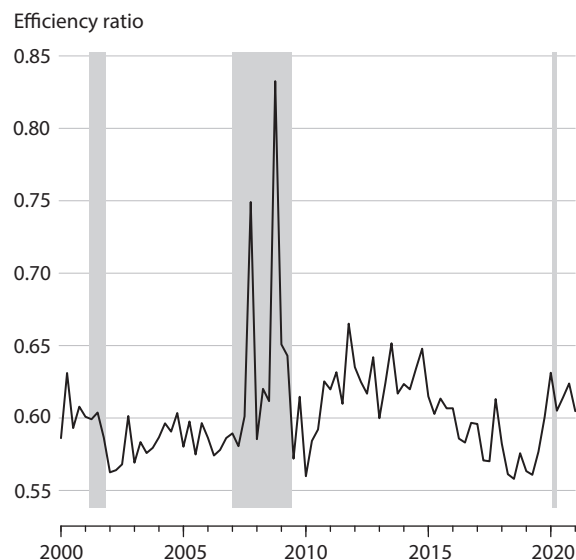
assets increased from 4.91 percent to 16.08 percent of total assets over the same period. The *Wall Street Journal* reported in January 2021 that “big banks have a tantalizing amount of cash on their books, but there is still a ways [sic] to go before that money can make its way into shareholders’ pockets” (Demos, 2021). The decline in net loans and leases and the increases in securities and cash assets held by banks during 2008-10 correspond loosely with periods of ultra-low interest rates discussed above. The fact that banks held more cash and lent less suggests that although the Federal Reserve can affect the supply of loans, it is more difficult for it to affect the demand for loans. Nonetheless, GDP has increased from 2000 to 2020.

Coincident with declining loans was an increase in bank deposits, as shown in Figure 12. Figure 12 shows that the quantity of deposits held by banks increased steadily after 2000 but began increasing at a much faster rate in the fourth quarter of 2019. The ratio of total deposits to total assets fluctuated around a flat trend until mid-2008, then increased sharply afterward, with large jumps in 2008-09 and in 2020, as shown in Figure 13. Perhaps even more dramatic is the increase in the ratio of total deposits to GDP shown in Figure 14. The ratio trended upward after 2000 but began to increase rapidly in the fourth quarter of 2019. The ratio of deposits to GDP increased by 0.9 percent in the third quarter of 2019, by 9.6 percent in the fourth quarter of 2019, and then by 18.6 percent in the first quarter of 2020 before falling by 7.0 percent in the second quarter of 2020 before increasing again in the second half of 2020.

A simple measure of banks’ performance is the ratio of non-interest expense to net interest income plus non-interest income, sometimes called “the efficiency ratio.” The ratio provides a

Figure 14**Ratio of Total Deposits Held by U.S. Commercial Banks to GDP**

NOTE: Gray bars indicate recessions as determined by the NBER.
SOURCE: FDIC (2021f) and U.S. Bureau of Economic Analysis (2021b).

Figure 15**Efficiency Ratio**

NOTE: The efficiency ratio is computed as non-interest expense divided by the sum of net interest income and non-interest income. Data are not seasonally adjusted. Gray bars indicate recessions as determined by the NBER.

SOURCE: FDIC (2021i,j,k).

measure of how well banks control their operating expenses; higher values imply more overhead in the form of labor, physical capital, and other expenses (apart from interest expense) relative to revenue. Seay and Tofiq (2020) report (in mid-December 2020) that U.S. banks' efficiency ratios are worsening, causing banks to focus on cost-cutting efforts. Figure 15 shows the efficiency ratio for the banking industry as a function of time. While there is some fluctuation from quarter to quarter, as Seay and Tofiq (2020) observe, the efficiency ratio has increased during the ongoing COVID-19 pandemic but has not reached the levels of the Great Recession. In fact, the efficiency ratio was higher at various points during 2011-14 than during recent quarters. Obviously, the efficiency ratio can increase if either non-interest expense increases or net interest income declines. Given the unusually low interest rates during 2011-14 and 2020, it seems that these low rates have caused the efficiency ratio to rise above levels experienced in other periods by reducing banks' net interest income.

As noted at the beginning of this section, the analysis here has focused on aggregate measures of banks' performance. The next section develops a model of bank production and employs statistical methods to estimate technical efficiency, providing further insight into how well banks convert financial and physical capital and labor into loans, securities, and off-balance sheet activities.

3 A MODEL OF BANKING PRODUCTION

The model described here is the model used by Simar and Wilson (2021) to develop nonparametric methods for estimating directional distance functions while avoiding endogeneity issues. Only a brief presentation is given here; for more discussion and details, see Simar and Wilson (2021). To establish notation, let $X \in \mathbb{R}_+^p$ and $Y \in \mathbb{R}_+^q$ denote stochastic vectors of input and output quantities, respectively, and let $x \in \mathbb{R}_+^p$ and $y \in \mathbb{R}_+^q$ denote fixed, nonstochastic vectors of input and output quantities, respectively. Let $f(x,y)$ denote the joint density of (X,Y) with support on a strict subset of \mathbb{R}_+^{p+q} and consider the production set

$$(3.1) \quad \Psi := \{(x, y) \mid x \text{ can produce } y\}$$

containing all feasible combinations of inputs and outputs. In the absence of any stochastic noise, the support of the joint density $f(x,y)$ is a subset of Ψ and $\Pr((X,Y) \in \Psi) = 1$. But if stochastic noise is present, then the support of $f(x,y)$ may extend outside Ψ and hence $\Pr((X,Y) \in \Psi) < 1$.

Assume that Ψ is a closed set, to ensure the existence of an efficient frontier, or technology, given by

$$(3.2) \quad \Psi^\partial := \{(x, y) \mid (\alpha^{-1}x, \alpha y) \notin \Psi \text{ for } \alpha > 1\}.$$

Now define $r := p + q$ and define a fixed, nonstochastic $(r \times 1)$ direction vector $d = (-d'_x, d'_y)'$, where $d_x \in \mathbb{R}_+^p$ and $d_y \in \mathbb{R}_+^q$. Then for a fixed point $(x, y) \in \mathbb{R}_+^r$, distance to the frontier Ψ^∂ in the direction d is given by the directional distance function

$$(3.3) \quad \delta(x, y \mid d) := \sup \{ \delta \mid (x - \delta d_x, y + \delta d_y) \in \Psi \}$$

whenever $\delta(x,y \mid d)$ exists.⁴

The model of Simar and Wilson (2021) assumes that a set of n independent, identically distributed (iid) optimal but latent points $W_i^\partial := (X_i^\partial, Y_i^\partial)$ on the efficient frontier Ψ^∂ are generated by a well-defined probability mechanism. These optimal points are unobserved due to inefficiency and statistical noise. Let $W_i := (X_i, Y_i)$ denote observed input-output combinations for $i = 1, \dots, n$ and assume that the random sample of observations $S_n = \{W_i\}_{i=1}^n$ is generated by the statistical generating process $W_i = W_i^\partial + \xi_i d$ so that

$$(3.4) \quad \begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} X_i^\partial \\ Y_i^\partial \end{bmatrix} + \xi_i d,$$

where, conditional on W_i^∂ , the ξ_i are independent scalar random variables whose characteristics may depend on W_i^∂ .

Next, the stochastic term ξ_i is assumed to consist of both noise and inefficiency. Specifically, assume that ξ_i includes both an inefficiency component $\eta_i \in \mathbb{R}_+$ and a noise component $\epsilon_i \in \mathbb{R}$ such that

$$(3.5) \quad \xi_i := \epsilon_i - \eta_i,$$

where conditional on W_i^∂ , ϵ_i and η_i are independent, with

$$(3.6) \quad \epsilon_i | W_i^\circ \sim \text{Sym} \left(0, \tilde{\sigma}_\epsilon \left(W_i^\circ \right) \right)$$

and

$$(3.7) \quad \eta_i | W_i^\circ \sim D_+ \left(\tilde{\sigma}_\eta \left(W_i^\circ \right) \right),$$

where $\text{Sym}(0, a)$ denotes a symmetric (around zero) two-sided distribution on \mathbb{R} with standard deviation $a \geq 0$, and $D_+(b)$ denotes a one-sided distribution on \mathbb{R}_+ belonging to some one-parameter scale family with parameter $b \geq 0$.⁵

As discussed by Simar and Wilson (2021), direct estimation of the proposed model involves significant complications related to endogeneity issues. Simar and Wilson (2021) propose transforming the model to a new space to avoid these difficulties. Let $V_d[v_1 \dots v_{r-1}]$ denote an orthonormal basis for the direction vector d (see, e.g., Noble and Daniel, 1977, for discussion and details). As noted by Simar and Wilson (2021), V_d is not unique, but this is not a problem as long as it is treated as fixed by using only one such matrix. Note that V_d is an $r \times (r-1)$ matrix and depends only on the given direction vector d . Now define the $(r \times r)$ rotation matrix

$$(3.8) \quad R_d = \begin{bmatrix} V_d' \\ d' / \|d\| \end{bmatrix},$$

where $\|\cdot\|$ denotes the L_2 norm.⁶

To transform the model, define

$$(3.9) \quad \begin{bmatrix} Z_i \\ U_i \end{bmatrix} := R_d W_i = R_d \begin{bmatrix} X_i \\ Y_i \end{bmatrix},$$

where $Z_i \in \mathbb{R}^{r-1}$ and $U_i \in \mathbb{R}^1$ for each $i = 1, \dots, n$, thereby transforming the iid sample S_n to an iid sample $S_n(d) = \{(Z_i, U_i)\}_{i=1}^n$. Clearly, $U_i = d' W_i / \|d\|$ and hence is invariant to the ordering of the inputs and outputs since the ordering of the elements of the direction vector d necessarily corresponds to whatever order is chosen for elements of the input and output vectors X_i and Y_i .

Note that the transformation from (x, y) -space to (z, u) -space is linear; it is also one to one and therefore can be inverted. The production set in (x, y) -space is transformed to the set

$$(3.10) \quad \Gamma_d = \left\{ (z, u) \in \mathbb{R}^r \mid R_d' [z' \ u'] \in \Psi \right\}$$

in (z, u) -space, and the density $f(x, y)$ in (x, y) -space is transformed to a density $g(z, u) = g(u|z)g(z)$ in (z, u) -space. More importantly, the frontier Ψ° —an $(r-1)$ -dimensional manifold in (x, y) -space—is transformed to the scalar-valued function $\phi(z) : \mathbb{R}_+^{r-1} \mapsto \mathbb{R}^1$ in (z, u) -space such that

$$(3.11) \quad \phi(z) = \sup \{ u \mid (z, u) \in \Gamma_d \}.$$

Pre-multiplying both sides of the model in (3.4) by the rotation matrix R_d yields the transformed model

$$(3.12) \quad \begin{bmatrix} Z_i \\ U_i \end{bmatrix} = \begin{bmatrix} Z_i^\circ \\ U_i^\circ \end{bmatrix} + \xi_i \begin{bmatrix} 0_{r-1} \\ \|d\| \end{bmatrix}.$$

Note that both U_i and $Z_i = Z_i^\partial$ are observed, given the direction vector d . Moreover, since $Z_i = Z_i^\partial$ is observed, the frontier points in (z,u) -space can be expressed as $(Z_i, U^\partial(Z_i)) = (Z_i, \phi(Z_i))$. Then by conditioning on Z_i ,

$$(3.13) \quad U_i = \phi(Z_i) + \|d\| \epsilon_i - \|d\| \eta_i,$$

where the scale functions $\bar{\sigma}_\epsilon(W_i^\partial)$ and $\bar{\sigma}_\eta(W_i^\partial)$ in (3.6) and (3.7) can be written as functions of Z_i due to the fact that W_i^∂ can be expressed as a function of Z_i only. Hence (3.6) and (3.7) can be rewritten as

$$(3.14) \quad \epsilon_i | Z_i \sim \text{Sym}(0, \sigma_\epsilon(Z_i))$$

and

$$(3.15) \quad \eta_i | Z_i \sim D_+(\sigma_\eta(Z_i)).$$

As shown by Simar and Wilson (2021), only U_i is endogenous in (3.12), making estimation much easier than working in the original (x,y) -space where all the elements of X_i and Y_i are potentially endogenous.

In order to identify the frontier function $\phi(Z_i)$ in (3.11), a functional form for the distribution D_+ in (3.15) must be specified, but no additional assumptions on the distribution of ϵ_i are needed. Assume that the distribution of η_i , conditional on Z_i , is half normal with scale parameter $\sigma_\eta(Z_i)$; that is,

$$(3.16) \quad \eta_i | Z_i \sim N^+(0, \sigma_\eta^+(Z_i)).$$

The scale parameter is functional and will be estimated locally as explained below. Consequently, the half-normal specification used here is rather flexible. Simar and Wilson (2021) present Monte Carlo results showing the effects of an incorrect specification for η , and while there is a price to pay for misspecification, it appears to be small since the estimation is local. Some additional mild technical assumptions regarding the smoothness of the frontier function $\phi(Z)$ and existence of moments of U_i are needed for consistency of the nonparametric estimators described in the next section; see Simar and Wilson (2021) for details. Such assumptions are far less restrictive than those imposed for parametric estimation.

4 ESTIMATION METHOD

Estimation of the transformed model is straightforward using the method proposed by Simar, Van Keilegom, and Zelenyuk (2017) and used by Simar and Wilson (2021). The goal is to obtain estimates of

$$(4.1) \quad \mu_\eta(Z_i) := E(\eta_i | Z_i) = k_1 \sigma_\eta(Z_i),$$

where $k_1 = 2^{1/2} \pi^{-1/2}$ due to the assumption of a local half-normal distribution for η .²

Note that $\mu_\eta(Z_i)$ defined in (4.1) must be nonnegative for all $i = 1, \dots, n$. In order to simplify notation, assume that the direction vector d is normalized so that $\|d\| = 1$ (this has no effect on the results that follow). Define $\varepsilon_i := \xi_i + \mu_\eta(Z_i)$ and $r_1(Z_i) := \phi(Z_i) - \mu_\eta(Z_i)$. Clearly, $E(\varepsilon_i | Z_i) = 0$, and it follows that

$$(4.2) \quad U_i = r_1(Z_i) + \varepsilon_i$$

and $E(U_i | Z_i) = r_1(Z_i) + \phi(Z_i) - \mu_\eta(Z_i)$. The function $r_1(Z_i)$ is an ordinary conditional mean function that can be estimated using standard nonparametric regression estimators (e.g., the local-linear estimator; see Wheelock and Wilson, 2001, 2018a, for examples).

By the assumption of symmetry for $\varepsilon_i | Z_i$,

$$(4.3) \quad E(\varepsilon | Z_i) = 0,$$

and

$$(4.4) \quad E(\varepsilon^3 | Z_i) = -E\left[(\eta - \mu_\eta(Z_i))^3 | Z_i\right].$$

Let $r_3(Z_i) := E(\varepsilon^3 | Z_i)$. Using the assumptions of the model, it is easy to show that

$$(4.5) \quad \mu_\eta(Z_i) = E(\eta | Z_i) = \sqrt{\frac{2}{\pi}} \sigma_\eta(Z_i)$$

and

$$(4.6) \quad r_3(Z_i) = \sqrt{\frac{2}{\pi}} \frac{\pi - 4}{\pi} \sigma_\eta^3(Z_i) \leq 0.$$

Due to (4.6), the scale function $\sigma_\eta(Z_i)$ is identified by the third (local) moment $r_3(Z_i)$, providing identification of $\mu_\eta(Z_i)$.

To estimate $\mu_\eta(Z_i)$, (4.2) is first estimated by local-linear least squares, yielding estimates $\hat{r}_1(Z_i)$ of $r_1(Z_i)$.⁸ Next, residuals $\hat{\varepsilon}_i = U_i - \hat{r}_1(Z_i)$ are computed for $i = 1, \dots, n$, and local-linear regression is used again to regress $\hat{\varepsilon}_i^3$ on Z_i to obtain estimates $\hat{r}_3(Z_i)$ of $r_3(Z_i)$.⁹

Substituting $\hat{r}_3(Z_i)$ into (4.6) yields (after some algebra) the estimator

$$(4.7) \quad \hat{\sigma}_\eta(Z_i) = \max \left\{ 0, \left[\sqrt{\frac{\pi}{2}} \frac{\pi}{\pi - 4} \hat{r}_3(Z_i) \right]^{1/3} \right\} \geq 0$$

of the scale function $\sigma_\eta(Z_i)$. Then substituting $\hat{\sigma}_\eta(Z_i)$ into (4.5) yields an estimator of mean conditional inefficiency $\mu_\eta(Z_i)$; that is,

$$(4.8) \quad \hat{\mu}_\eta(Z_i) = \sqrt{\frac{2}{\pi}} \hat{\sigma}_\eta(Z_i).$$

The only remaining issues surround specification of specific inputs and outputs, and this is discussed in the next section.

5 DATA AND VARIABLE SPECIFICATION

As noted in Section 1, there is a large literature on estimation of efficiency among banks. The specification of inputs and outputs used here is rather standard and similar to the variable specifications used by Wheelock and Wilson (2012, 2018a). Five inputs are specified, including purchased funds (X_1), consisting of deposits, federal funds purchased and securities sold under agreements to repurchase, trading liabilities, other borrowed money, and subordinated notes and debentures; labor (X_2), measured in full-time equivalents; physical capital (X_3), measured as the book value of premises and fixed assets, including capitalized leases; equity (X_4); and nonperforming loans (X_5), including loans past due 30-89 days and still accruing, loans past due 90 days or more and still accruing, loans not accruing, and other real estate owned. Both purchased funds and equity are sources of financial capital. Nonperforming loans are included to provide a measure of risk-taking, which is otherwise difficult to measure. Other real estate owned is included in this measure to reflect foreclosed loans. In the analysis that follows, both X_4 and X_5 are held constant when estimating technical efficiency.¹⁰ Three outputs are specified: (i) total loans and leases held for investment and held for sale (Y_1); (ii) securities (Y_2), including held-to-maturity securities, available-for-sale debt securities, equity securities with readily determinable fair values not held for trading, and federal funds sold and securities purchased under agreements to resell; and (iii) off-balance-sheet activities (Y_3). Securities represent a type of lending other than loans. Off-balance sheet activities are measured here by non-interest income and include loan commitments, letters of credit, revolving underwriting facilities, and other activities that potentially generate revenue.¹¹ All dollar amounts are in thousands of constant 2012 dollars, and labor is measured in terms of full-time equivalents.

Data on commercial banks are taken from the quarterly Federal Financial Institutions Examination Council call reports for commercial banks for the first quarter of 2000 through the fourth quarter of 2020. After deleting outliers and observations with missing or obviously invalid values, 518,165 observations remain, covering the 84 quarters from the first quarter of 2000 through the fourth quarter of 2020. The numbers of observations in each quarter range from 8,236 in the first quarter of 2000 to 4,152 in the fourth quarter of 2020, reflecting the decreasing numbers of U.S. banks, as discussed earlier in Section 2. The first eight rows of Table 1 present summary statistics on the input and output variables. The values shown for the first, second, and third quartiles, as well as for comparison of the median and mean values, reveal considerable right-skewness in the marginal distributions, reflecting similar skewness in the size distribution of U.S. banks.

With $p = 5$ inputs and $q = 3$ outputs, there are potentially seven right-hand-side variables in the nonparametric regressions that must be estimated in order to obtain estimates of technical efficiency. Moreover, it is well known that nonparametric estimators such as local-linear least squares are subject to the “curse of dimensionality,” which means that their convergence rates become slower with increasing dimensionality. This translates into increasing estimation error as the number of right-hand-side variables increases in nonparametric regressions. However, both the input variables and the output variables are highly collinear, allowing the dimensionality of the problem to be reduced using the methods examined by Wilson (2018).

To employ dimension reduction, each of the input and output variables defined above are first divided by their standard deviations. Next, let X_1 denote the $(n \times 3)$ matrix whose columns contain observations on X_1 , X_2 , and X_3 . Similarly, let X_2 denote the $(n \times 2)$ matrix whose columns contain observations on X_4 and X_5 , and let Y represent the $(n \times 3)$ matrix whose columns contain observa-

Table 1**Summary Statistics for Inputs and Outputs**

	Min	Quartile 1	Median	Mean	Quartile 3	Max
X_1	2.7183(2)	5.9195(4)	1.2189(5)	8.2012(5)	2.7449(5)	1.6834(9)
X_2	1.0000	1.8500(1)	3.6000(1)	1.7132(2)	7.8500(1)	2.3482(5)
X_3	8.6466(-1)	7.5831(2)	2.2583(3)	1.1090(4)	5.7194(3)	1.1321(7)
X_4	0.0000	1.2866(4)	2.8746(4)	2.2939(5)	6.6601(4)	9.5467(8)
X_5	0.0000	2.8400(2)	7.5600(2)	1.3913(4)	2.2120(3)	4.1728(7)
Y_1	1.6680(2)	7.3079(3)	1.4484(4)	9.9085(4)	3.1371(4)	1.9245(8)
Y_2	0.0000	3.4929(1)	5.4878(2)	1.0017(4)	2.4543(3)	6.6467(7)
Y_3	4.0000	3.5380(4)	8.0414(4)	5.4528(5)	1.9270(5)	9.3292(8)
X_1^*	6.5958(-5)	4.0802(-3)	8.3114(-3)	6.0068(-2)	1.8684(-2)	1.9145(2)
X_2^*	4.5063(-4)	8.1962(-3)	1.7908(-2)	8.9313(-2)	4.0639(-2)	1.1473(2)
Y^*	3.5902(-5)	3.5545(-3)	7.5067(-3)	6.0806(-2)	1.7562(-2)	1.4207(2)
Z_1	-7.3519	-4.2017	-3.6583	-3.5481	-3.0396	4.0130
Z_2	-3.9117	2.6067	3.2320	3.1371	3.7934	6.1151
U	-2.9426(3)	-1.5689	-6.8551(-1)	-3.0313	-3.0085(-1)	9.8881(1)

NOTE: All monetary values are in thousands of constant 2012 dollars. Values are represented in scientific notation with the number $a \times 10^b$ denoted as $a(b)$; for example, the number 1.1824×10^2 is denoted as 1.1824(2).

tions on the three output variables. Then an eigensystem decomposition is performed to obtain eigenvalues and eigenvectors of the moment matrices $X_1'X_1$, $X_2'X_2$, and $Y'Y$. For each moment matrix, the ratio of the largest eigenvalue to the sum of eigenvalues for the moment matrix gives a measure of the portion of the independent linear information contained in the first principal component (e.g., for X_1 , the first principal component is the $(n \times 1)$ vector $X_1^* = X_1\Lambda_{X_1}$, where Λ_{X_1} is the (3×1) eigenvector corresponding to the largest eigenvalue of $X_1'X_1$). Computing this ratio for each moment matrix yields values 0.9523, 0.8091, and 0.9118 for $X_1'X_1$, $X_2'X_2$, and $Y'Y$, respectively. These values indicate that most of the information in X_1 , X_2 , and Y is contained in their first principal components, and hence the principal components are used for estimation. Summary statistics for the principal components X_1^* , X_2^* , and Y^* are shown in rows 9 to 11 of Table 7.¹²

Using the principal components X_1^* , X_2^* , and Y^* results in $p = 2$, $q = 1$, and hence $r = 3$ in terms of the notation of Sections 3 and 4. Thus the direction vector d contains three elements. The first element is set equal to the negative of the median of X_1^* , the second element (corresponding to X_2^*) is set to zero, and the third element is set equal to the median of Y^* . Setting the second element of the direction vector equal to zero ensures that distance to the frontier is measured while holding constant both equity and the risk measure given by nonperforming loans. The chosen direction vector completely determines the rotation matrix R_d , as explained in Section 3. Applying the rotation matrix to X_1^* , X_2^* , and Y^* yields the transformed variables Z_1 , Z_2 , and U , as described in Section 3. Summary statistics for these transformed variables appear in the last three rows of Table 1.

6 ESTIMATION RESULTS

The data used for estimation form an unbalanced panel covering 84 quarters. Consequently, both (fixed) time and firm effects are included in the regression of U on Z and the regression of cubed residuals from this regression on Z . For estimation of $r_1(\cdot)$ in (4.2), U_i is regressed on (Z_i, T_i, L_i) , $i = 1, \dots, n$, where $T_i \in \{1, 2, \dots, 84\}$ (corresponding to the 84 quarters represented in the sample), and L_i is a unique numeric label for the firm represented by observation i . Local-linear least-squares regression amounts to feasible generalized least squares with the $(n \times n)$ weighting matrix consisting of a diagonal matrix of kernel weights. A Gaussian product kernel is used for the two continuous elements of Z_i . For estimation of $r_\ell(Z_i, T_i, L_i)$, $\ell \in \{1, 3\}$, at observation i the j th diagonal element of the weighting matrix is given by

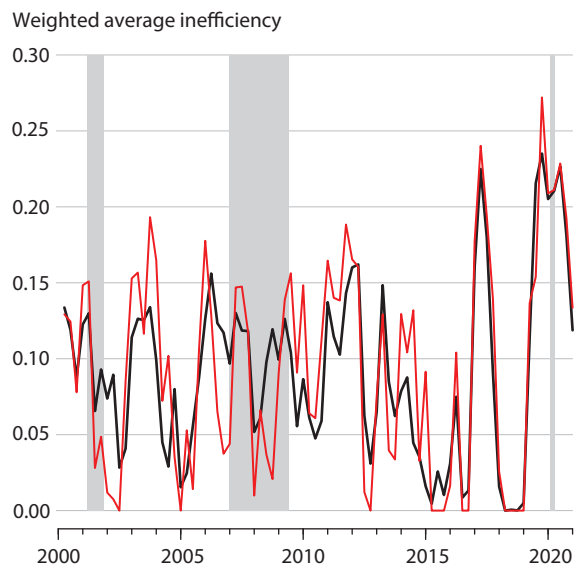
$$(6.1) \quad \omega_j^{(\ell)} = \left[\prod_{k=1}^2 \left(h_k^{(\ell)} \right)^{-1} K \left(\frac{Z_{jk} - Z_{ik}}{h_k^{(\ell)}} \right) \right] \left(h_3^{(\ell)} \right)^{|T_j - T_i|} \left(h_4^{(\ell)} \right)^{I(L_j \neq L_i)},$$

where Z_{ik} denotes the k th element of Z_i , $K(\cdot)$ denotes a kernel function (i.e., the standard normal density function), $I(\cdot)$ denotes the indicator function, and $h_k^{(\ell)}$, $k \in \{1, \dots, 4\}$ are bandwidths with $h_k^{(\ell)} \in [0, 1]$ for $k \in \{3, 4\}$. The rate of convergence in the nonparametric regressions depends on the number of continuous right-hand-side variables (two in our case) and is not affected by the discrete time and firm effects (see Li and Racine, 2007, or Henderson and Parmeter, 2015, for discussion). Smaller (larger) values of the bandwidths result in less (more) smoothing or more (less) localization. Bandwidths are optimized via leave-one-out, least-squares cross validation, which amounts to choosing values for bandwidths to minimize a least-squares estimate of the integrated mean-square error. See Simar and Wilson (2021) for details and additional discussion.

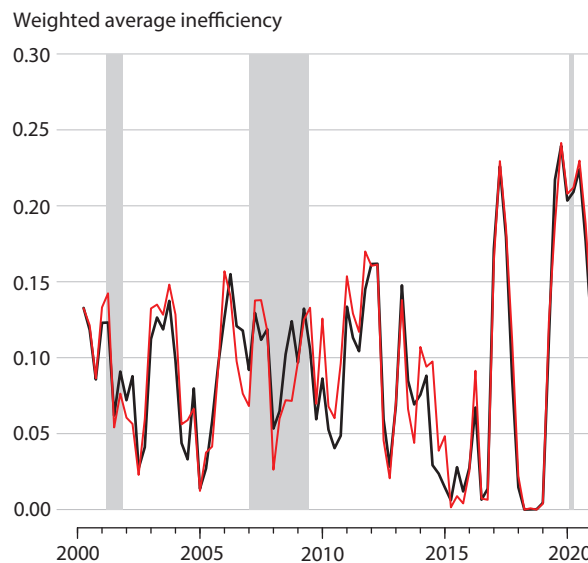
For each observation in the sample, estimates $\sigma_\eta(Z_i)$ are computed using (4.7), and these are used to compute estimates $\hat{\mu}_\eta(Z_i)$ using (4.8). The estimates $\hat{\mu}_\eta(Z_i)$ give the *expected* inefficiency for observation i facing Z_i . Then for each quarter, weighted averages of the $\hat{\mu}_\eta(Z_i)$ are computed for (i) the five largest banks in each quarter and (ii) all other banks in each quarter, with the weight for bank i given by its total assets in the given quarter. These weighted averages are plotted in Figure 16, where the red curve corresponds to the five largest banks (i.e., the top 5), and the black curve corresponds to all other banks, with larger (smaller) values indicating greater (less) technical inefficiency. Figure 17 shows similar information for the top 50 and non-top 50 banks in each quarter.

It is evident that the two curves in Figure 16 and in Figure 17 follow a similar pattern. However, where peaks occur in Figure 16, the top 5 banks are typically more inefficient on average banks outside the top 5. The same is true for the top 50 versus non-top 50 banks in Figure 17, but the differences are smaller than in Figure 16. At the same time, when average inefficiency is low, the top 5 (and top 50) banks are often less inefficient than the non-top 5 (and non-top 50) banks. Consistent with the discussion in Section 2, the 2001 recession seems to have had little adverse impact on banks' technical inefficiency. Average inefficiency peaked at the beginning of the brief 2001 recession but was lower at the end of the recession.

The estimates in both figures reveal fluctuations in technical efficiency, as does the efficiency ratio in Figure 15, with the fluctuations in technical efficiency becoming larger beginning in 2017. Both Figures 16 and 17 show technical efficiency initially improving during the Great Recession but then worsening again at the end of the recession.

Figure 16**Weighted-Average Technical Efficiency Estimates, Top 5 Banks Versus All Others**

NOTE: The red curve shows averages of technical efficiency estimates for the five largest (top 5) banks weighted by total assets in each period. The black curve shows similar values for non-top 5 banks. Smaller (larger) values correspond to greater (less) efficiency. Gray bars indicate recessions as determined by the NBER.

Figure 17**Weighted-Average Technical Efficiency Estimates, Top 50 Banks Versus All Others**

NOTE: The red curve shows averages of technical efficiency estimates for the 50 largest (top 50) banks weighted by total assets in each period. The black curve shows similar values for non-top 50 banks. Smaller (larger) values correspond to greater (less) efficiency. Gray bars indicate recessions as determined by the NBER.

While Figure 15 shows the efficiency ratio increasing during the Great Recession, Figures 16 and 17 show technical efficiency improving during the Great Recession but then worsening again at the end of the recession. This contrasts with the efficiency ratio depicted in Figure 15, where large peaks occur during the Great Recession. The efficiency ratio gives a rough measure of profit efficiency, whereas technical inefficiency estimates do not consider cost nor revenue, but instead consider resource usage and output production. The two measures are not unrelated but can (and sometimes do) deviate.

Both Figures 16 and 17 show technical inefficiency peaking prior to the 2020 pandemic-induced recession but then falling rapidly after reaching the peak. A large peak in technical inefficiency also occurs in the third quarter of 2017 as interest rates were climbing, but in both cases, the estimates suggest that banks quickly recovered. The efficiency ratio plotted in Figure 15 shows a similar, though less pronounced, pattern.

The large peaks in technical inefficiency during 2017 and 2019-20 shown in Figures 16 and 17 are consistent with the data on deposits and net loans and leases discussed in Section 2. As discussed earlier and shown in Figure 12 and as reported in the financial press (e.g., *Financial Review*, 2020, and Son, 2020), an unusually rapid and large increase in deposits occurred beginning in the fourth quarter of 2019, while loans increased much less. The model developed in Section 3 treats deposits as an input and loans as an output, and consequently technical inefficiency is found to increase to high levels in 2019 and 2020. The efficiency ratio plotted in Figure 15 includes only interest income

and non-interest income and expense, and so was less affected than technical inefficiency by the rapid increase in deposits.

7 CONCLUSIONS

The technical efficiency estimates discussed in Section 6 provide an additional measure of banks' performance, beyond examination of specific items such as loans and deposits from banks' balance sheets. The estimates of technical efficiency indicate, among other things, that while banks experience periods of high technical inefficiency, these episodes are typically short-lived; that is, while banks may experience periods of inefficiency, they typically recover quickly, at least on average. The efficiency ratio shown in Figure 15 reached higher levels in the Great Recession of 2007-09 than in the short recession of 2020, but technical inefficiency estimates are found to be higher on average in 2019-20 than in 2007-09. Periods of high technical inefficiency imply that financial resources are being wasted, but the cost of this waste depends not only on the quantities involved, but also on prices, that is, interest rates.

Of course, the recession of 2007-09 was much longer than the recession of 2020, making comparisons problematic. However, 25 commercial banks failed in 2008, followed by 140 in 2009 and then 157, 92, and 51 in 2010, 2011, and 2012, respectively. By contrast, only four banks failed in 2019, another four in 2020, and none in 2021.¹³ Wheelock and Wilson (2000) find evidence that cost and technical inefficiency contribute to the probability of bank failure. However, extrapolating from the experience of the Great Recession, it may be that while technical inefficiency reached high levels in 2019 and 2020, the episode was of short duration and did not have time to lead to another wave of insolvencies among banks. To the extent that the Fed's stimulus policy during the pandemic period limited the duration of the 2020 recession, it is likely that a number of banks remained solvent that might otherwise have failed. ■

NOTES

- ¹ Data on the number of institutions are from the Federal Deposit Insurance Corporation (FDIC, 2021a). Data on failures and assisted mergers are also from the FDIC (2021h). The FDIC resolves bank failures by arranging mergers of failed institutions with other banks; such mergers are referred to as "assisted mergers."
- ² NBER recession data are available at <https://www.nber.org/research/data/us-business-cycle-expansions-and-contractions>, and the recession dates determined by the NBER are used in the figures. Dating recessions in the economy necessarily involves some subjectivity. See the *FRED® Blog* (2021) for additional discussion.
- ³ Throughout, dollar values are expressed in constant 2012 U.S. dollars.
- ⁴ Directional distance functions were first proposed by Chamber, Chung, and Färe (1996, 1998). It is possible that the path $(x, y) + \delta d$ does not pass through Ψ for any real-valued δ , in which case $\delta(x, y|d)$ does not exist.
- ⁵ A density $f(\cdot)$ belongs to a one-parameter scale family if it can be written as $f(\cdot) = (1/\sigma)\tilde{f}(\cdot/\sigma)$ for some $\sigma > 0$, where $\tilde{f}(\cdot)$ is any density on \mathbb{R}_+ . Examples include the half-normal and exponential distributions and the gamma and Weibull distributions with fixed-shape parameters.
- ⁶ See Noble and Daniel (1977) or Goldstein (1980) for discussion and explanation of rotation matrices.
- ⁷ Using a different distributional assumption would change the value of k_1 .
- ⁸ See Fan and Gijbels (1996) for details. The properties of this estimator are well known; see Fan and Gijbels (1996) or Li and Racine (2007) for discussion.

- ⁹ Simar, Van Keilegom, and Zelenyuk (2017) show that the usual asymptotic properties (i.e., consistency, rates of convergence, and asymptotic normality) for $\hat{r}_3(Z_i)$ hold under mild regularity conditions. See Simar and Wilson (2021) for additional discussion.
- ¹⁰ Equity is difficult to adjust in the short run. Risk is held constant in order to evaluate banks' performance at observed levels of risk-taking.
- ¹¹ See FDIC (2021I, Section 3.8) for discussion and additional details.
- ¹² The extensive simulation results reported by Wilson (2018) suggest that when efficiency is estimated using nonparametric free-disposal hull or data envelopment analysis estimators, estimation based on the first principal components of X_1 , X_2 , and Y (denoted by X_1^* , X_2^* , and Y^*) is likely to result in less estimation error than would arise using the original eight variables. To my knowledge, no comprehensive Monte Carlo experiments have been done to provide guidelines for when dimension reduction should be used with local-linear estimation, but the results of Wilson (2018) are suggestive. More work is needed.
- ¹³ Data on bank failures are from the FDIC (2021g).

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