Discussion of
Lars Svensson and Noah Williams
"Bayesian and Adaptive Optimal Policy Under Model Uncertainty."

James Bullard

Federal Reserve Bank of St. Louis

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- Why?

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  - This just creates a new model.
- It is not clear that the meta-model concept is the right one for thinking about "model uncertainty."

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- Markov jump-linear-quadratic system, with forward-looking variables.
- "Modes," or regimes, follow a Markov process.
- Difference from previous work: modes not observable here.
- Both private sector and policymakers infer modes from their observations of the economy.

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- That system would have a (Bayesian) rational expectations equilibrium.
- That equilibrium may or may not be learnable in standard senses.

Related to MJLQ?

- The authors study equations suggested by an engineering literature:

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\begin{aligned}
X_{t+1} &= A_{11,j+1}X_t + A_{12,j+1}x_t + B_{1,j+1}i_t + C_{1,j+1}\epsilon_{t+1} \quad (1) \\
E_t H_{j,t+1} &= A_{21,j,t}X_t + A_{22,j,t}x_t + B_{2,j,t}i_t + C_{2,j,t}\epsilon_t \quad (2)
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- The matrices would not switch like this: instead, agents would take actions based on their inferred probability of being in any particular mode.

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- Put the emphasis on learning and inference about the probability distribution of modes.
- Assume all agents update their perceived distribution over modes.
- The "REE" occurs when the subjective perceived probability distribution of modes coincides with the exogenous, true distribution.
- Optimal policy takes into account that the distribution of modes is initially unknown.

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- Stability conditions for this process? Expectational stability or related concept?
- How is the REE obtained?
- Some versions may induce a self-confirming equilibrium? Especially when policy choices affect observed modes.

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- Size of experimentation motive drives differences between BOP and AOP.

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- Do the beliefs of the policymaker make sense? Possibly they think modes are independently drawn.

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- Perceived versus true transition equation: Under what conditions do they coincide?
- Expectational stability may be a concern.

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- Cannot ask that question here.

Conclusions

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- BOP only works for small models due to curse of dimensionality.
- AOP a computable alternative, but stability is an open question.
- Remark: There is much greater model uncertainty in the world than is acknowledged in this paper.