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 "December 2, 2001";
"E STABILITY OF LAGGED DATA INERTIAL RULES";
"General case with phix, phipi, phir all positive";

Clear[phix, phipi, phir, sigma, kappa, rho, beta, capb1, bbar, omega, delta, bbar,
 bbar11, bbar12, bbar13, bbar21, bbar22, bbar23, bbar31, bbar32, bbar33]

<< LinearAlgebra`MatrixManipulation`;

(* Matrices for MSV solution and E-stability*);

omega = {{1, sigma, 0},
    {kappa, kappa * sigma + beta, 0},
  {0, 0, 0}};

delta = {{-phix * sigma, -phipi * sigma, -phir * sigma},
    {-kappa * phix * sigma, -kappa * phipi * sigma, -kappa * sigma * phir},
  {phix, phipi, phir}};

"bbar matrix below";
bbar = {{bbar11, bbar12, bbar13},
   {bbar21, bbar22, bbar23},
 {bbar31, bbar32, bbar33}};

"Relations in MSV solution for bbar matrix";

bbar11 = (phix / phir) * bbar13;
bbar21 = (phix / phir) * bbar23;
bbar12 = (phipi / phir) * bbar13;
bbar22 = (phipi / phir) * bbar23;
bbar31 = phix;
bbar32 = phipi;
bbar33 = phir;

"function '
  kronecker' defines how to compute the kronecker product of 2 matrices";

kronecker[f_, p_List, q_List] :=
 Flatten[Map[Flatten, Transpose[Outer[f, p, q], {1, 3, 2}], {2}], 1];

"E STABILITY CONDITIONS GIVEN BELOW";

DT1[a_] := kronecker[Times, Transpose[a], omega] +
  kronecker[Times, IdentityMatrix[3], omega . a];
DT2[a_] := rho * omega + omega . a;
DT3[a_] := omega + omega . a;

mat3 = Simplify[omega + omega . bbar];
mat3i = mat3 - IdentityMatrix[3];
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"CHARACTERISTIC POLYNOMIAL OF  (omega+omega.bbar- I) COMPUTED BELOW";
chpmat3i = Simplify[-Det[mat3i - mu * IdentityMatrix[3]]]

"characteristic polynomial of (omega+omega.bbar- I) below";
chpmat3i = -1/phir ((1 + mu) (bbar13 (kappa (1 + mu) phipi + (1 - beta + mu) phix) +
      phir (-mu + beta mu - mu^2 + kappa sigma + kappa mu sigma) +
      bbar23 (beta mu phipi + kappa phipi sigma +
         kappa mu phipi sigma + phix sigma + mu phix sigma)));

"one eigenvalue of chpmat3i is -1 and
  other two are given by the characteristic polynomial chp3ii";

chp3ii = mu^2 - bbar13 kappa phipi/phir - bbar13 phix/phir + bbar13 beta phix/phir -
  kappa sigma - bbar23 kappa phipi sigma/phir - bbar23 phix sigma/phir +
  mu (1 - beta - bbar23 beta phipi/phir - bbar13 kappa phipi/phir - bbar13 phix/phir -
     kappa sigma - bbar23 kappa phipi sigma/phir - bbar23 phix sigma/phir);

mat1 = Simplify[DT1[bbar] - IdentityMatrix[9]];

"CHARACTERISTIC POLYNOMIAL OF  (DT1[bbar]-I) COMPUTED BELOW";
chpmat1 = Simplify[-Det[mat1 - mu * IdentityMatrix[9]]]

"characteristic polynomial of (DT1[bbar]- I) evaluated below";
chpmat1 = 1/phir^4 ((1 + mu)^5 (- (1 + mu) phir + bbar13 (kappa phipi + phix) +
       bbar23 (beta phipi + (kappa phipi + phix) sigma)) ^2
    (2 bbar23^2 beta phipi^2 +
      2 bbar13^2 beta phix^2 + phir^2 (1 + 2 mu + mu^2 - phir - beta phir - mu phir -
         beta mu phir + beta phir^2 - kappa phir sigma - kappa mu phir sigma) -
      bbar13 phir (kappa phipi + kappa mu phipi + 2 phix + beta phix + 2 mu phix +
         beta mu phix - 3 beta phir phix + kappa phix sigma + kappa mu phix sigma) +
      bbar23 (- (1 + mu) phir phix sigma +
         phipi (-phir - 2 beta phir - mu phir - 2 beta mu phir + 3 beta phir^2 +
            4 bbar13 beta phix - 2 kappa phir sigma - 2 kappa mu phir sigma))));

mat2 = Simplify[rho * omega + omega . bbar];
mat2i = mat2 - IdentityMatrix[3];

"CHARACTERISTIC POLYNOMIAL OF  (rho*omega+omega.bbar- I) COMPUTED BELOW";
chpmat2ii = Simplify[-Det[mat2i - mu * IdentityMatrix[3]]]

"characteristic polynomial of (rho*omega+omega.bbar- I) below";
chpmat2i =
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$$-\frac{1}{\text{phir}}\,((1+\text{mu})\,(\text{bbar13}\,(\text{kappa phipi}+\text{kappa mu phipi}+\text{phix}+\text{mu phix}-\text{beta phix rho})$$
$$\text{bbar23}\,(\text{beta phipi}\,(1+\text{mu}-\text{rho})+(1+\text{mu})\,(\text{kappa phipi}+\text{phix})\,\text{sigma})+$$
$$\text{phir}\,(-1-2\,\text{mu}-\text{mu}^2+\text{rho}+\text{beta rho}+\text{mu rho}+$$
$$\text{beta mu rho}-\text{beta rho}^2+\text{kappa rho sigma}+\text{kappa mu rho sigma})));$$

"Simplified expression for final 2 eigenvalues of (rho*omega+
omega.bbar- I) given by following characteristic polynomial";

$$1+\text{mu}^2-\frac{\text{bbar23 beta phipi}}{\text{phir}}-$$
$$\frac{\text{bbar13 kappa phipi}}{\text{phir}}-\frac{\text{bbar13 phix}}{\text{phir}}-\text{rho}-\text{beta rho}+\frac{\text{bbar23 beta phipi rho}}{\text{phir}}+$$
$$\frac{\text{bbar13 beta phix rho}}{\text{phir}}+\text{beta rho}^2-\frac{\text{bbar23 kappa phipi sigma}}{\text{phir}}-\frac{\text{bbar23 phix sigma}}{\text{phir}}-$$
$$\text{kappa rho sigma}+\text{mu}\left(2-\frac{\text{bbar23 beta phipi}}{\text{phir}}-\frac{\text{bbar13 kappa phipi}}{\text{phir}}-\frac{\text{bbar13 phix}}{\text{phir}}-\right.$$
$$\left.\text{rho}-\text{beta rho}-\frac{\text{bbar23}\,(\text{kappa phipi}+\text{phix})\,\text{sigma}}{\text{phir}}-\text{kappa rho sigma}\right);$$