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Output Cycles under Two Alternative Financial Systems

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Abstract

Different financial systems vary in the way they contribute to the process of resource allocation in the economy and in the risk-sharing pattern that they bring about. It would therefore be plausible to expect different financial systems to differ in the way they affect real economic activity. I hereby provide a theoretic framework for the comparison and analysis of output cycles under two alternative financial systems: an equity-based financial system (EFS), in which a mutual fund functions as a financial intermediary, versus a debt-based financial system (DFS), in which a bank plays that role. The research points that DFS generates larger output cycles and a higher expected output than EFS. The mechanism that generates these results is the counter-cyclical effect of savings' behavior under EFS.

[1] I am indebted to the late Prof. Oved Yosha and to Prof. Alex Cukierman for their kind academic guidance. I would also like to thank Prof. Assaf Razin and Prof. Jean Tirole for usefull insights. Errors are mine.

1. Introduction

Financial systems play an essential role in the process of resource allocation in an economy. They transform household savings into investment funds for the corporate sector, thus providing the former a means for intertemporal consumption smoothing and enabling the later to carry out investment and production plans. Different financial systems vary in the way they carry out these functions, and in the risk-sharing pattern that they bring about. It would therefore be plausible to expect different financial systems to differ in the way they affect real economic activity. This paper provides a theoretic framework for the analysis and comparison of output cycles under two alternative financial systems: an equity-based financial system (EFS) versus a debt-based financial system (DFS).

The EFS versus DFS debate is by no means a new one, and has given rise to a large body of literature. Among the contributors to this debate are authors such as Fama (1980), Goodhart (1993) and Miller (1998), who advance the view that EFS outdo DFS in terms of financial stability; Allen & Gale (1997), Bhattacharya, Fulghieri and Rovelli (1998), who compare the performance of different financial systems in achieving efficient intertemporal allocation of resources, and Allen & Gale (2000) - whose wide-ranging book provides a comprehensive survey of works in the realm. Nevertheless, the real macroeconomic activity implications of financial systems have, to the best of my knowledge, so far remained untreated.

As reported by Allen & Gale (2000), the current trend is that of moving towards market-oriented financial systems. The rapidly changing technological and regulatory environment gives rise to alternatives to banks, which have traditionally dominated the financial arena. For firms, financial markets' instruments such as equity, commercial papers and corporate bonds provide a substitute to bank loans. For households, mutual funds, especially money market mutual funds (MMMFs), provide an alternative to bank deposits. Since their first appearance

in the United States in the early '70s, MMMFs have exhibited substantial market share growth. In her empirical study, Whiting (1994) argues that "…MMMFs now acquired share that would conceivably have gone to the banking institutions". In assessing the desirability of this ongoing trend and of financial markets' reforms, it would be important to acquire a better understanding of the way by which different types of financial systems affect real economic activity and business cycles.

This paper provides a theoretic framework for the analysis and comparison of output cycles under two alternative financial systems: an equity-based mutual funds' banking system versus a debt-based commercial banking system. I present a non-monetary, three-period model of an economy consisting of risk-averse households, risk-neutral entrepreneurs, and a competitive, risk-neutral financial intermediary. Households seek to secure future consumption through savings, whereas entrepreneurs seek to procure external funding in order to start up projects. Projects are subjected to both aggregate and idiosyncratic uncertainties that affect their risk of facing external funding constraints. Projects that face external funding constraints expire, while others, who succeed in procuring external funding, survive and generate positive returns. Two versions of the model are then developed: an equity-based financial system (EFS) version, in which the financial intermediary takes the form of a mutual fund, and a debt-based financial system (DFS) version, in which it takes the form of a bank. The mutual fund offers state-contingent thrift contracts to the households and equity-based, state-contingent funding contracts to the corporate sector; the bank, on the other hand, offers fixed-term deposit contracts to the households and debt-based, state-contingent loan contracts to the corporate sector[2]. It can therefore be seen that EFS brings about some degree of risk sharing in the economy, whereas DFS isolates households form corporate sector's risks. It is then shown that EFS induces savings to behave in a counter-cyclical

[2] In the real world, mutual funds' assets and liabilities vary in the same line, whereas banks assets are usually more volatile than their liabilities.

manner, whereas DFS neither mitigates nor empowers exogenous cyclical forces. The paper points that the economy demonstrates milder output fluctuations under EFS than under DFS. It is important to note, however, that this result may be sensitive to the assumption regarding the respective bargaining power of entrepreneurs and the financial intermediary - a point to be hopefully examined in future research. The paper is organized as follows: sections 2 and 3 present the equity and debt versions of the model, respectively; section 4 analyzes and compares the solutions for the model's two versions, and finally, section 5 summarizes and highlights the main results.

2. An Equity-Based Model

Consider a small non-monetary economy consisting of entrepreneurs, households and a mutual fund. There are three periods ($t = 0, 1, 2$) and a single, non-storable good that serves as both capital and consumption good. All agents behave competitively and attribute the same importance to the different periods.

Production in the economy is carried out through projects. A typical project requires an initial investment of one unit of capital in period 0 and matures two periods later. The project's quality is unknown in period 0 and is publicly revealed in period 1 as either good or bad. A good quality project will generate an output of $R > 1$ units of capital in period 2. A bad quality project undergoes financial distress in period 1: it will turn unfruitful unless an immediate, crucial investment of $\alpha < R$ units of capital allows it to generate an output of R units of capital in period 2[3]. Liquidation of a project prior to period 2 is assumed unattainable. Let $\tilde{\alpha}^n$ stand for the quality contingent investment required by project n in period 1; let $\tilde{q}^n$ be the random quality variable of project n that takes the values 0 (good) and 1 (bad), then:

[3] The gross rate return is therefore R for a good-quality project and $\frac{R}{1+\alpha}$ for a bad-quality project that manages to procure α units of capital in period 1.

(1) $\tilde{\alpha}^n = \alpha\tilde{q}^n \quad , \quad \tilde{q}^n \in \{0,1\}$

The following production function summarizes the ex-ante technology of project n:

$$(2) \quad f(k^n_0, k^n_1) \equiv \begin{cases} R & \text{if } k^n_0 = 1 \text{ and } k^n_1 \geq \tilde{\alpha}^n \\ \mathbf{0} & \text{if } k^n_0 = 1 \text{ and } k^n_1 < \tilde{\alpha}^n \text{ or } k^n_0 < 1 \end{cases}$$

where k^n_0 , k^n_1 are the amounts of capital invested in project n in periods 0 and 1 respectively. In addition to the idiosyncratic uncertainty concerning the quality of each project, agents are subjected to aggregate uncertainty regarding the state of the world in the economy. Let $\tilde{z}$ be a state of the world random variable that is resolved in period 1 as either h (high) or ℓ (low) with probabilities θ and 1-θ respectively. The state of the world determines the fraction of good projects, $\pi(\tilde{z})$, and that of bad projects, 1-$\pi(\tilde{z})$. State h is characterized by a higher proportion of good projects (and by the same token, a lower proportion of bad projects) with respect to state ℓ, so that $0 < \pi(\ell) < \pi(h) < 1$. The ex-ante compound probability that a project will reveal as being of a good quality is $\bar{\pi} \equiv \theta\pi(h) + (1-\theta)\pi(\ell)$; accordingly, the complementary compound probability that the project will reveal as being of a bad quality is $1-\bar{\pi} \equiv \theta[1-\pi(h)] + (1-\theta)[1-\pi(l)]$. Figure 1 provides a schematic illustration of a typical project's production process.

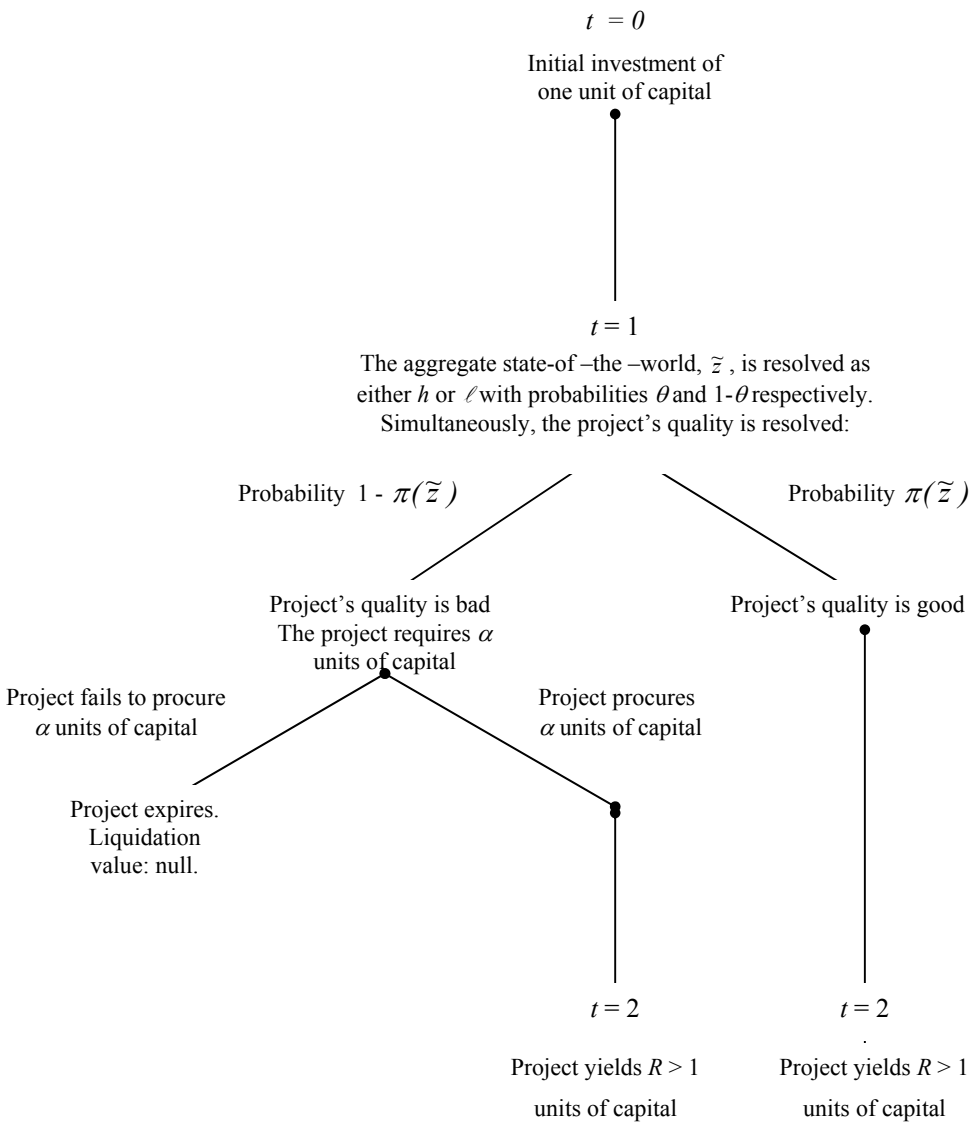
2.1 The Agents

2.1.1 Entrepreneurs

There is a continuum of competitive entrepreneurs on the [0 , 1) interval, each endowed with 0 < ε < 1 units of capital in period 0. A typical entrepreneur is risk-neutral and aims to maximize his period 2 consumption level. In period 0, each entrepreneur can engage in a single project. As a project requires an initial investment of one unit of capital in period 0, the

entrepreneur will need a complementary external funding of 1-ε units of capital to start up his project.

Figure 1. A typical projects' production process



The corporate sector can obtain external funding from the mutual fund in exchange of corporate shares. In period 0, entrepreneur n seeks to procure 1-ε units of capital by giving away a share $0 \leq \phi_0^n \leq 1$ of his project to the mutual fund. In case his project undergoes

financial distress in period 1, the entrepreneur will seek to procure the crucial investment of α units of capital by augmenting the fund's total share in the project to $\phi_1^n(\tilde{z})$ so that $\phi_0^n \leq \phi_1^n(\tilde{z}) \leq 1$. Therefore, if entrepreneur n succeeds in procuring the additional α units of capital, he will eventually hold 1- $\phi_1^n(\tilde{z})$ of his project.

In periods 0 and 1 the fund does not necessarily provide all the projects in the economy with the funding they need. The fraction of projects that eventually procure external funding in any given period depends upon the capital inflow to the fund, the funding needs of the projects, the expected return on the projects' shares and the fund's opportunity cost for providing the projects with funding. Let $0 \leq \lambda_0 \leq 1$ be the fraction of projects in the economy that are provided with external funding in period 0; a fraction 1- λ_0 of the projects are therefore financially constrained and cannot be established. In the same manner, let $0 \leq \lambda_1(\tilde{z}) \leq 1$ stand for the state-dependent fraction of financially distressed projects that succeed in procuring α units of capital from the fund in period 1 and therefore, survive. A fraction 1- $\lambda_1(\tilde{z})$ of financially distressed projects face external funding constraints and therefore expire.

2.1.2 The Households

The economy is populated by a unity of competitive, risk-averse households, each endowed with γ_0 units of capital in period 0 and γ_1 units of capital in period 1, where $0 < \gamma_0 < 1$, $0 < \gamma_1 < 1$. Household i's utility from periodical consumption is captured by the utility function $u(c^i_t) = ln(c^i_t)$, where c^i_t stands for household i's consumption level in period t. Households cannot borrow; however, the mutual fund allows them to save in periods 0 and 1 and thus, reallocate their resources. In period 0, the fund offers the households long-term saving contracts, yielding a state-contingent gross return rate $r_0(\tilde{z})$ within two periods. In period 1, the fund offers the households short-term saving contracts, yielding a state-

contingent gross return rate $r_1(\tilde{z})$ within one period. Let s_0^i and $s_1^i(\tilde{z})$ represent the amounts of capital saved by household i in period 0 and period 1 state $\tilde{z}$, $\tilde{z} \in \{h, \ell\}$ respectively; then, in period 0, the household traces its state-contingent lifetime consumption path by solving:

$$(3)\quad \{c_0^i, \{c_t^i(\tilde{z})\}_{t=1}^2\} = argmax\{ln(c_0^i) + \theta\, ln[c_1^i(h)] + (1-\theta)\, ln[c_1^i(\ell)] + \theta\, ln[c_2^i(h)] + (1-\theta)\, ln[c_2^i(\ell)]\}$$

s.t.

$$(3a)\quad c_0^i + s_0^i \leq \gamma_0$$

$$(3b)\quad c_1^i(\tilde{z}) + s_1^i(\tilde{z}) \leq \gamma_1 \quad , \quad \tilde{z} \in \{h, \ell\}$$

$$(3c)\quad c_2^i(\tilde{z}) \leq s_0^i r_0(\tilde{z}) + s_1^i(\tilde{z}) r_1(\tilde{z}) \quad , \quad \tilde{z} \in \{h, \ell\}$$

$$(3d)\quad s_0^i \geq 0$$

$$(3e)\quad s_1^i(\tilde{z}) \geq 0 \quad , \quad \tilde{z} \in \{h, \ell\}$$

Restrictions (3a) – (3c) are the household's budget constraints whereas restrictions (3d) and (3e) stem from the household's inability to borrow. Substituting (3a)-(3c) in (3) and solving for s_0^i, $s_1^i(h)$ and $s_1^i(\ell)$ yields the following first-order conditions:

$$(4)\quad \frac{1}{\gamma_0 - s_0^i} = \frac{\theta r_0(h)}{s_0^i r_0(h) + s_1^i(h) r_1(h)} + \frac{(1-\theta) r_0(\ell)}{s_0^i r_0(\ell) + s_1^i(\ell) r_1(\ell)}$$

$$(5)\quad \frac{1}{\gamma_1 - s_1^i(\tilde{z})} = \frac{r_1(\tilde{z})}{s_0^i r_0(\tilde{z}) + s_1^i(\tilde{z}) r_1(\tilde{z})} \quad , \quad \tilde{z} \in \{h, \ell\}$$

Equation (4) gives the first-order condition for s_0^i, household i's savings in period 0; it states that the marginal utility from consumption in period 0 equals the expected utility from consumption in period 2. Equation (5) gives the first-order condition for $s_1^i(\tilde{z})$, household i's

state-dependent savings in period 1; it states that for any given state of the world, the marginal utility from consumption in period 1 equals the marginal utility from consumption in period 2.

2.1.3 The Mutual Fund

There is one competitive, risk-neutral mutual fund in the economy. The fund bears null operation costs and therefore does not charge any fees. In periods 0 and 1, the fund uses its liabilities - households' savings - to build up its asset portfolio. Two types of assets are available to the fund: corporate shares, being the state-contingent liabilities of projects to the fund, and alternative riskless assets[4]. The terms of both types of assets are the same. In period 0, alternative assets are long-term assets that yield a gross rate of return of one within two periods; in period 1, alternative assets are short-term assets that yield a gross return rate of one in period 2. It is assumed that there exist no assets other than corporate shares and alternative riskless assets, and that the fund is the only agent in the economy that can purchase these types of assets. The alternative assets could be given several interpretations. They could represent foreign assets, government bonds or some capital storage technology that is available to the fund alone (a volt, for instance).

The fund's objective is to maximize its assets' value in any given period. Being risk-neutral it will therefore purchase, in any given period, the assets whose expected yields are the highest. It is assumed that the fund favors corporate shares over alternative assets yielding the same expected return.

Figure 2 summarizes the course of actions in the model:

Figure 2. The model's time-line

$t = 0$	$t = 1$ Aggregate and idiosyncratic uncertainties are resolved	$t = 2$
• Households consume and make long term savings (s_0); • Entrepreneurs need external funding of $1 - \varepsilon$ units of capital each to start up their projects; • The fund uses households' pooled savings to engage in long-term investments; thus a fraction λ_0 of entrepreneurs is granted funding.	• Households consume and make short-term state dependent savings ($s_1(\tilde{z})$); • Bad-quality projects require funding of α units of capital each to survive; • The fund uses households pooled savings to engage in short-term investments; thus a fraction $\lambda_1(\tilde{z})$ of the bad-quality projects obtain extra funding.	• Projects mature and yield R units of capital each; • The fund distributes redeems its shares in the projects and pays returns to the households. • Households use the returns to consume.

2.2 Equilibrium Conditions

This section presents the conditions for a symmetric nash-equilibrium of the model. Given that entrepreneurs are ex-ante identical and households are identical, a symmetric equilibrium of the equity-version of the model is characterized by ex-ante identical projects and identical households' savings' levels. This is captured by the following equations:

(6a) $\phi_0^n = \phi_0 \qquad \forall\, n$

(6b) $\phi_1^n(h) = \phi_1(h) \qquad \forall\, n$

(6c) $\phi_1^n(\ell) = \phi_1(\ell) \qquad \forall\, n$

[4] The reason for the inclusion of alternative assets is to provide the fund with an opportunity cost to purchasing corporate assets.

(6d) $s_0^i = s_0 \qquad \forall\, i$

(6e) $s_1^i(h) = s_1(h) \qquad \forall\, i$

(6f) $s_1^i(\ell) = s_1(\ell) \qquad \forall\, i$

Equation (6a) states that in period 0, all the projects in the economy offer the mutual fund the same share in exchange for external funding. Equations (6b) and (6c) state that for a given state of the world, all the projects that undergo financial distress in period 1 offer the fund the same augmented share in return of an extra investment of α units of capital. Equation (6d) states that in equilibrium, all households save the same amount of capital in period 0. Equations (6e) and (6f) state that for a given state-of –the-world, all households save the same amount of capital in period 1.

It is assumed that entrepreneurs enjoy full bargaining power with respect to the fund[5] (this assumption is necessary to obtain a workable solution). Given (6a)-(6c), the fund's equilibrium stipulation for providing an entrepreneur with the external funding he needs in period 0 is:

(7) $[\theta\pi(h)+(1-\theta)\pi(\ell)]\phi_0 R+\theta[1-\pi(h)]\lambda_1(h)\,[\,\phi_1(h)R-\alpha\,] + (1-\theta)[1-\pi(\ell)]\lambda_1(\ell)\,[\,\phi_1(\ell)R-\alpha]$

$\geq\ 1-\varepsilon$

The right hand of condition (7) is the expected gross return to the fund from purchasing a share ϕ_0 of a typical domestic project in period 0. The fund takes into account that the project might undergo financial distress in period 1, a case in which the fund may salvage it in exchange for a larger share $\phi_1(\tilde{z})$ or let it expire. The left hand of (7) is the opportunity cost of investing 1-ε units of capital in a typical project in period 0, embodied by the gross return

[5] This assumption enables to obtain a workable solution of the model. In general, if the funds' bargaining power is high, it might find it unworthy to supply capital to the corporate sector. Assuming a more balanced distribution of bargaining power between the entrepreneur and the fund would call for the use numeric methods in order to solve the model, which at this stage I would rather avoid.

that the fund could obtain by investing the same amount of capital in alternative, riskless assets. Therefore, the fund would be willing to venture 1-ε units of capital in a project in period 0 if the expected gross return it thus obtains is not inferior to the riskless gross return it could secure by purchasing alternative assets with the same amount of capital. In equilibrium, due to the competitive behavior of entrepreneurs and the fund, condition (7) holds with strict equality. It follows that in equilibrium, corporate shares and alternative assets offer the fund the same expected gross return in period 0. It is therefore obvious why alternative assets are needed: unless there were alternative assets, the fund would obtain null equilibrium gross return from corporate assets. In such case, the fund would deliver null returns to the households so that the households would be deprived of a thrift device.

The fund's stipulation for investing α units of capital in a financially distressed project in period 1 would be:

(8) $\phi_1(\tilde{z})R \geq \alpha \quad , \quad \tilde{z} \in \{h, \ell\}$

The left hand of condition (8) is the fund's state-dependent return from salvaging a financially distressed project. The right hand of (8) is the fund's opportunity cost for doing so – the gross return it could obtain by investing α units of capital in alternative riskless assets. Condition (8) states that in period 1, the fund would be willing to salvage a distressed project if the gross return it thus obtains is not inferior to that offered by alternative assets. In equilibrium, due to the competitive behavior of entrepreneurs and the fund, condition (8) holds with strict equality; corporate shares and alternative assets therefore offer the fund the same gross return in period 1.

Another feature of equilibrium is that the total inflow of capital to the fund equals the total outflow of capital from the fund in any given period. In period 0, the households save s_0 units of capital that are placed in the fund. Let B_0 represent the units of capital invested by the fund in long-term alternative assets in period 0, then the equilibrium condition is:

(9) $s_0 = \lambda_0(1 - \varepsilon) + B_0$

The right hand of (9) is the total inflow of capital to the fund in period 0, whereas the left hand of (9) is the total outflow of capital from the fund in that period, consisting of investments in corporate shares and alternative assets. Investment in domestic assets in period 0 sums up to $\lambda_0(1-\varepsilon)$ units of capital: the fund provides a fraction λ_0 of the entrepreneurs in the economy with $1-\varepsilon$ units of capital, the external funding that each needs to start up his project. Rearranging (9) gives:

(10) $\lambda_0 = \dfrac{s_0 - B_0}{1-\varepsilon}$

Equation (10) presents the fraction of projects that eventually manage to procure external funding in period 0 as the ratio between the fund's capital supply to the corporate sector and the corporate sector's aggregate capital demand. In period 1, the state-dependent capital inflow to the fund is $s_1(\tilde{z})$. The fund's state-dependent capital outflow in period 1 consists of the purchase of financially distressed projects' shares and of alternative assets. In equilibrium, total capital inflow equals total capital outflow in period 1, for any given state of the world. Therefore:

(11) $s_1(\tilde{z}) = [1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})\alpha + B_1(\tilde{z}) \quad , \quad \tilde{z} \in \{h, \ell\}$

The right hand of equation (11) is the fund's total state-dependent capital outflow in period 1**,** consisting of a total investment of $I_1(\tilde{z})$ units of capital. Total investments in period 1 consist of $[1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})\alpha$ units of capital that are invested in financially distressed projects and $B_1(\tilde{z})$ units of capital that are invested in short-term alternative assets. Rearranging (11) yields:

(12) $\lambda_1(\tilde{z}) = \dfrac{s_1(\tilde{z}) - B_1(\tilde{z})}{[1-\pi(\tilde{z})]\lambda_0\alpha} \quad , \quad \tilde{z} \in \{h, \ell\}$

The nominator in the right side of equation (12) is the fund's capital supply to the domestic corporate sector in period 1: the fund's total investments in period 1 minus its purchase of new alternative assets. The denominator in the right side of equation (12) is the financially distressed projects' aggregate demand for external funding in period 1. Therefore, the fraction of financially distressed projects that are salvaged is the ratio between the fund's capital supply to the domestic corporate sector and the domestic corporate sector's aggregate capital demand.

Another feature of equilibrium is that the fund makes zero profits, regardless of the state-of-the-world. The zero-profit condition means that in equilibrium, the fund does not keep for itself any part of the capital placed in it by the households. Let $B_1(\tilde{z})$ be the state-dependent amount of capital invested by the fund in short-term alternative assets in period 1; then, the fund's zero-profit equilibrium condition in period 1 is the following:

$$(13)\quad s_0\frac{r_0(\tilde{z})}{r_1(\tilde{z})}+s_1(\tilde{z}) = \pi(\tilde{z})\lambda_0 R\phi_0+[1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})\phi_1(\tilde{z})R \ +B_0+B_1(\tilde{z}) \ , \ \tilde{z}\in\{h,\ell\}$$

The left hand of (13) is households' total savings' value in period 1 state $\tilde{z}$, that is, the fund's total liabilities at that time. The right hand of equation (13) is the fund's total investments value in period 1 state $\tilde{z}$, that is, its total assets at that time: a share ϕ_0 in $\pi(\tilde{z})\lambda_0$ good projects, a share $\phi_1(\tilde{z})$ in $[1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})$ bad projects and a total investment of $B_0+B_1(\tilde{z})$ units of capital in alternative assets.

In period 2, the value of the fund's state-dependent liabilities to the households becomes $r_0(\tilde{z})s_0+r_1(\tilde{z})s_1(\tilde{z})$ units of capital. However, as the fund does not purchase additional assets in the concluding period, and since all uncertainties were resolved in period 1, the value of the fund's assets remains the same as it was in period 1. Therefore, the fund's zero-profits equilibrium condition in period 2 is the following:

(14) $r_0(\tilde{z})s_0 + r_1(\tilde{z})s_1(\tilde{z}) = \pi(\tilde{z})\lambda_0 R\phi_0 + [1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})\phi_1(\tilde{z})R + B_0 + B_1(\tilde{z}) \;,\; \tilde{z} \in \{h, \ell\}$

Since the fund's assets' value in period 2 is the same as in period 1, the equilibrium value of the fund's liabilities in period 1 is the same as in the beginning of period 2. Substituting (14) in (13) yields:

(15) $r_1(\tilde{z}) = 1 \quad , \qquad \tilde{z} \in \{h, \ell\}$

Equation (15) states that the equilibrium gross return rate offered by the fund on households' savings in period 1 equals one, regardless of the prevailing state of the world. This is perfectly intuitive: since the fund does not charge any fees, the return on the assets it purchases in a given period equals the return gained by the households on the capital they placed at the fund at the same period. As the equilibrium gross return rate on the assets purchased by the fund in period 1 is one, so will be the gross return rate on period 1 savings. Given (15), conditions (13) and (14) are reduced to the following single zero-profits condition:

(16) $r_0(\tilde{z})s_0 + s_1(\tilde{z}) = \pi(\tilde{z})\lambda_0 R\phi_0 + [1-\pi(\tilde{z})]\lambda_0\lambda_1(\tilde{z})\phi_1(\tilde{z})R + B_0 + B_1(\tilde{z}) \;,\; \tilde{z} \in \{h, \ell\}$

In equilibrium, corporate shares offer the fund in period 0 an expected return that is equal to the riskless return offered by long-term alternative assets. As it is assumed that the fund favors corporate shares over alternative assets yielding the same return, it follows that:

(17) $B_0 = max\{ s_0 - 1 + \varepsilon \;,\; 0 \}$

Equation (17) states that in equilibrium, the fund will purchase alternative assets in period 0 only once the corporate sector's financial needs are fully satisfied. In equilibrium, corporate shares of bad-quality salvaged projects and short-term alternative assets yield the fund the same riskless return. As the fund favors corporate shares over alternative assets yielding the same return, it follows that:

(18) $B_1(\tilde{z}) = max\{ s_1(\tilde{z}) - [1-\pi(\tilde{z})]\lambda_0\alpha \;,\; 0 \} \quad , \quad \tilde{z} \in \{h, \ell\}$

Equation (18) states that in equilibrium, the fund will purchase short-term alternative assets in period 1 only once the financial needs of financially distressed projects have been satisfied.

3. A Debt-Based Model

In this version of the model, a competitive risk-neutral bank plays the role of financial intermediary. The bank is characterized by fixed liabilities and state-contingent assets, as opposed to the mutual fund in the former version of the model, whose both assets and liabilities were state-contingent.

3.1 The Agents

3.1.1 Entrepreneurs

In period 0, the bank offers the corporate sector long-term loans, bearing a state-contingent gross interest rate $\rho_0(\tilde{z})$. In period 1, given the materialized state of the world, the bank grants the corporate sector short-term loans, bearing a fixed gross interest rate of $\rho_1(\tilde{z})$.

3.1.2 The Households

The bank offers the households fix-term saving contracts in periods 0 and 1 alike. In period 0, it provides the households with a long term saving contract, bearing a fixed gross interest rate r_0. One period later, given the realized state of the world, the bank offers the households a short-term saving contract bearing a gross interest rate $r(\tilde{z})$. The typical household traces its optimal state-contingent lifetime consumption path by solving:

(19) $$\{c_0^i, \{c_t^i(\tilde{z})\}_{t=1}^2\} = argmax\{ln(c_0^i) + \theta ln[c_1^i(h)] + (1-\theta)ln[c_1^i(\ell)] + \theta ln[c_2^i(h)] + (1-\theta)ln[c_2^i(\ell)]\}$$

s.t.

(19a) $c_0^i + s_0^i \leq \gamma_0$

(19b) $c_1^i(\tilde{z}) + s_1^i(\tilde{z}) \leq \gamma_1 \quad , \quad \tilde{z} \in \{h, \ell\}$

(19c) $c_2^i(\tilde{z}) \leq s_0^i r_0 + s_1^i(\tilde{z}) r_1(\tilde{z}) \quad , \quad \tilde{z} \in \{h, \ell\}$

(19d) $0 < s_0^i < \gamma_0$

(19e) $0 \leq s_1^i(\tilde{z}) < \gamma_1 \quad , \quad \tilde{z} \in \{h, \ell\}$

Substituting (19a)-(19c) in (19) and solving for s_0^i, $s_1^i(h)$ and $s_1^i(\ell)$ yields the following first-order conditions:

(20) $$\frac{1}{\gamma_0 - s_0^i} = \frac{\theta r_0}{s_0^i r_0 + s_1^i(h) r_1(h)} + \frac{(1-\theta) r_0}{s_0^i r_0 + s_1^i(\ell) r_1(\ell)}$$

(21) $$\frac{1}{\gamma_1 - s_1^i(\tilde{z})} = \frac{r_1(\tilde{z})}{s_0^i r_0 + s_1^i(\tilde{z}) r_1(\tilde{z})} \quad , \quad \tilde{z} \in \{h, \ell\}$$

Equation (20) gives the first-order condition for s_0^i, the household's savings in period 0; it states that the marginal utility from consumption in period 0 equals the expected utility from consumption in period 2. Equation (21) gives the first-order condition for $s_1^i(\tilde{z})$, the household's state-dependent savings in period 1; it states that for any given state of the world, the marginal utility from consumption in period 1 equals the marginal utility from consumption in period 2.

3.2 Equilibrium Conditions

This section specifies the symmetric equilibrium conditions of the debt version of the model. Since households in the economy are identical, a symmetric equilibrium of the model is characterized by households' identical saving decisions, that is, equations (6d)-(6f) apply to the debt version of the model as well.

The bank's stipulation for granting an entrepreneur with a loan of 1-ε units of capital in period 0 is:

(22) $\theta\pi(h)(1-\varepsilon)\rho_0(h)+(1-\theta)\pi(\ell)(1-\varepsilon)\rho_0(\ell)+\theta[1-\pi(h)]\lambda_1(h)[(1-\varepsilon)\rho_0(h)+\alpha\rho_1(h)-\alpha]$

$+(1-\theta)[1-\pi(\ell)]\lambda_1(\ell)[(1-\varepsilon)\rho_0(\ell)+\alpha\rho_1(\ell)-\alpha] \geq 1-\varepsilon$

The left side of condition (22) is the expected pay back of the project to the bank in period 2. The bank takes into account the possibility that in any given state of nature, the project might undergo financial distress in period 1 - a case in which the bank may grant it an additional loan of α units of capital, or, alternatively, let it expire. The right side of (22) is the bank's opportunity cost for lending an entrepreneur 1-ε units of capital in period 0, taking the form of the riskless gross return it can obtain by investing that amount of capital in long-term foreign assets. Condition (22) therefore states that the bank would be willing to lend an entrepreneur 1-ε units of capital in period 0 if the expected payback of the loan is not inferior to the opportunity cost of extending it. In equilibrium, due to the competitive behavior of entrepreneurs and the bank, condition (22) holds with strict equality.

The bank's stipulation for lending α units of capital to a financially distressed project in period 1 is:

(23) $(1-\varepsilon)\rho_0(\tilde{z}) + \alpha\rho_1(\tilde{z}) \geq \alpha \quad , \quad \tilde{z} \in \{h, \ell\}$

By extending the project a short-term loan of α units of capital in period 1, the bank guarantees itself the payback of the initial long-term loan and, in addition, is certain about the payback of the additional short-term loan; this is captured by the left side of equation (23). The right side of equation (23) is the opportunity cost of extending the project the additional short-term loan, embodied by the gross return that the bank can obtain by investing α units of capital in short-term foreign assets. In equilibrium, due to the competitive behavior of the entrepreneurs and the bank, condition (23) holds with strict equality. Note that $\rho_1(\tilde{z})$ can be

negative; a negative $\rho_1(\tilde{z})$ is interpreted as the bank's willingness to discharge a bad project part of its debts in state $\tilde{z}$.

Other features of equilibrium are that (1) the capital inflow to the bank equals the capital outflow from the bank in any given period and state-of-the-world and (2) the bank favors a domestic asset over a foreign asset yielding the same return. It follows that equations (9)-(12) and (17)-(18) hold in the bank version of the model as well.

Like the fund in the equity-based model, the bank makes zero profits in equilibrium, that is, it does not keep for itself any part of households' savings and/or assets' returns. In period 1, the bank's zero-profit equilibrium condition is:

$$(24) \quad s_0 \frac{r_0}{r_1(\tilde{z})} + s_1(\tilde{z}) = \lambda_0 \pi(\tilde{z})(1-\varepsilon)\rho_0(\tilde{z}) + \lambda_0[1-\pi(\tilde{z})]\lambda_1(\tilde{z})[(1-\varepsilon)\rho_0(\tilde{z}) + \alpha\rho_1(\tilde{z})] + B_0 + B_1(\tilde{z}) \quad , \quad \tilde{z} \in \{h, \ell\}$$

The right hand of (24) is the bank's total asset's value in period 1, state $\tilde{z} \in \{h, \ell\}$.In the beginning of period 2, the value of the bank's state-dependent liabilities to the households is $s_0 r_0 + s_1(\tilde{z}) r_1(\tilde{z})$. Since the bank does not purchase any additional assets in period 2 and as all uncertainties were resolved in period 1, the value of the bank's assets remains the same as it was in period 1. Therefore, the bank's zero-profit equilibrium condition in period 2 is:

$$(25) \quad s_0 r_0 + s_1(\tilde{z}) r_1(\tilde{z}) = \lambda_0 \pi(\tilde{z})(1-\varepsilon)\rho_0(\tilde{z}) + \lambda_0[1-\pi(\tilde{z})]\lambda_1(\tilde{z})[(1-\varepsilon)\rho_0(\tilde{z}) + \alpha\rho_1(\tilde{z})] + B_0 + B_1(\tilde{z}) \quad , \quad \tilde{z} \in \{h, \ell\}$$

Since the bank's assets' value in period 2 is the same as in period 1, the equilibrium value of the bank's liabilities in period 1 is the same as in period 2. Substituting (24) in (25) yields:

$$(26) \quad r_1(\tilde{z}) = 1 \quad , \quad \tilde{z} \in \{h, \ell\}$$

Equation (26) is, in fact, identical to equation (15) in the equity version of the model; it states that the equilibrium gross return rate offered by the bank on households' savings in period 1

equals one, regardless of the prevailing state of the world. Given (26), equations (24) and (25) are reduced to the following single equation:

(27) $s_0 r_0 + s_1(\tilde{z}) = \lambda_0 \pi(\tilde{z})(1-\varepsilon)\rho_0(\tilde{z}) + \lambda_0[1-\pi(\tilde{z})]\lambda_1(\tilde{z})[(1-\varepsilon)\rho_0(\tilde{z}) + \alpha\rho_1(\tilde{z})] + B_0 + B_1(\tilde{z})$

$\tilde{z} \in \{h, \ell\}$

4. Equity versus Debt: A Comparison

This section presents, analyses and compares the outcomes of the equity and debt versions of the model.

4.1 Equilibrium Solutions

A prerequisite for the existence of equilibrium under both versions of the model is that the households' optimal savings level in period 0 is strictly positive, that is, $s_0 > 0$. To achieve considerable simplification, the equilibrium solutions that are presented for both versions of the model are confined to internal equilibria, that is, equilibria in which $s_1(\tilde{z}) > 0$, $0 < \lambda_0 < 1$ and $0 < \lambda_1(\tilde{z}) < 1$ for $\tilde{z} \in \{h, \ell\}$. The equilibrium levels of the variables in the equity and debt versions of the model are denoted by superscripts *E* and *D* respectively.

4.1.1 Solution for the Equity Version

The equity version of the model has two potential internal equilibria, stemming from two potential solutions for s_0^E. As the equilibrium analytical solutions for s_0^E are far too complicated to present, a full analytical equilibrium solution of the equity version of the model is unattainable. The equilibrium solution for s_0^E is such that satisfies equation (28a). Equations

(28b) – (28j) present the internal equilibrium solutions for the rest of the endogenous variables in terms of parameters and s_0^E.

(28a) $3\pi(h)\pi(\ell)s_0^{E2} + \{\gamma_1\bar{\pi}[\pi(h)+\pi(\ell)+2\bar{\pi}] - 2\gamma_0\pi(h)\pi(\ell)\}s_0^E + \gamma_1\bar{\pi}^2(\gamma_1 - 2\gamma_0) = 0$

where $0 < s_0^E < \gamma_0$

(28b) $s_1^E(\tilde{z}) = \dfrac{\gamma_1}{2} - \dfrac{\pi(\tilde{z})s_0^E}{2\bar{\pi}}$

(28c) $r_0^E(\tilde{z}) = \dfrac{\pi(\tilde{z})}{\bar{\pi}}$

(28d) $r_1^E(\tilde{z}) \equiv r_1^E = 1 \quad , \quad \tilde{z} \in \{h, \ell\}$

(28e) $\phi_0^E = \dfrac{1-\varepsilon}{\bar{\pi}R}$

(28f) $\phi_1^E(\tilde{z}) \equiv \phi_1^E = \dfrac{\alpha}{R} \quad , \quad \tilde{z} \in \{h, \ell\}$

(28g) $\lambda_0^E = \dfrac{s_0^E}{1-\varepsilon}$

(28h) $\lambda_1^E(\tilde{z}) = \dfrac{[\bar{\pi}\gamma_1 - \pi(\tilde{z})s_0^E](1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(\tilde{z})]s_0^E}$

(28i) $B_0^E = 0$

(28j) $B_1^E(\tilde{z}) \equiv B_1^E = 0 \quad , \quad \tilde{z} \in \{h, \ell\}$

Lemma 1. s_0^E *has only one internal solution: the positive root of equation (28a).*

Proof. See the Appendix.

The straightforward implication of lemma 1 is that the equity-based model has a unique internal equilibrium.

4.1.2 Solution for the Debt-Based Model

The debt-model has only one potential equilibrium. Equations (29a)-(29j) present the analytical equilibrium solution of the debt-based model:

(29a) $s_0^D = \dfrac{2\gamma_0 - \gamma_1}{3} \quad where \quad s_0^D > 0$

(29b) $s_1^D(\tilde{z}) \equiv s_1^D = \dfrac{2\gamma_1 - \gamma_0}{3} \quad , \quad \tilde{z} \in \{h, \ell\}$

(29c) $r_0^D = 1$

(29d) $r_1^D(\tilde{z}) \equiv r_1^D = 1 \quad , \quad \tilde{z} \in \{h, \ell\}$

(29e) $\rho_0^D(\tilde{z}) = \dfrac{1}{\pi(\tilde{z})}$

(29f) $\rho_1^D(\tilde{z}) = 1 - \dfrac{1 - \varepsilon}{\alpha\pi(\tilde{z})}$

(29g) $\lambda_0^D = \dfrac{2\gamma_0 - \gamma_1}{3(1 - \varepsilon)}$

(29h) $\lambda_1^D(\tilde{z}) = \dfrac{(2\gamma_1 - \gamma_0)(1 - \varepsilon)}{\alpha(2\gamma_0 - \gamma_1)[1 - \pi(\tilde{z})]}$

(29i) $B_0^D = 0$

(29j) $B_1^D(\tilde{z}) \equiv B_1^D = 0 \quad , \quad \tilde{z} \in \{h, \ell\}$

4.2 Outcome Comparison

Lemma 2. *The conditions for the existence of internal equilibria for both the equity and the debt versions of the model are:*

i. $\varepsilon \geq 1 - \alpha\bar{\pi}$;

ii. $\dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)]+\pi(h)(1-\varepsilon)} < s_0^E < \min\{1-\varepsilon\ ,\ \dfrac{\bar{\pi}\gamma_1}{\pi(h)}\}$;

iii. $\max\{\dfrac{\gamma_0}{2}\ ,\ 2\gamma_0 - 3(1-\varepsilon)\} < \gamma_1 < \dfrac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}$

iv. *if* $\gamma_1 > \dfrac{\pi(h)(1-\varepsilon)}{\bar{\pi}}$ *then the following condition replaces condition iii:*

$$\max\{\frac{\gamma_0}{2}\ ,\ 2\gamma_0 - 3(1-\varepsilon)\} < \gamma_1 < \min\{\frac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}\ ,\ \alpha[2-\pi(h)]\}$$

v. *if* $\gamma_0 > 2(1-\varepsilon)$, *than an additional condition is that* $\gamma_0 < 2(1-\varepsilon)+\alpha[1-\pi(h)]$

Proof. See the Appendix

4.2.1 Savings' Behavior

Proposition 1.

i. *Long-term savings are higher under DFS than under EFS, that is,* $s_0^E < s_0^D$.

ii. *Let* β *denote the degree of similitude between the two potential aggregate states of the world, so that*

$$\beta \equiv \frac{\pi(\ell)}{\pi(h)} \quad , \quad 0 \leq \beta < 1$$

then s_0^D *is invariant to* β *whereas* s_0^E *is increasing in* β *and*

$$s_0^E \xrightarrow[\beta\to 0]{} \frac{3\theta}{1+2\theta} s_0^D \qquad , \qquad s_0^E \xrightarrow[\beta\to 1]{} s_0^D$$

iii. s_0^D *is invariant to* θ *whereas* s_0^E *is decreasing in* θ *for* $\theta < \hat{\theta}$ *and increasing in* θ *for* $\theta > \hat{\theta}$*, where* $\hat{\theta}$ *is some threshold value of* θ *such that* $\hat{\theta} < \frac{1}{2}$*. In addition:*

$$s_0^E \xrightarrow[\theta \to 1]{} s_0^D \quad , \quad s_0^E \xrightarrow[\theta \to 0]{} s_0^D$$

Proof. See the Appendix.

The proofs for the results in proposition 1 are purely technical and rather intricate; however, all three results simply reflect the negative effect of risk on long-term savings. DFS insulates households from technological aggregate risk by granting a fixed rate of return r_0^D=1 on long-term savings. EFS, by contrast, offers state-contingent rates of returns on long-term savings - $r_0^E(h)$ and $r_0^E(\ell)$ - thus inducing households to provide the corporate sector partial insurance against aggregate technological risk. Equations (28c) and (29c) indicate that the expected rate of return on long-term savings under EFS equals the fixed rate of return on long-term savings under DFS:

(30) $\theta r_0^E(h) + (1-\theta) r_0^E(\ell) = 1 = r_0^D$

The risk households are subjected to under EFS, reflected by uncertainty as to the potential rates of return on long-term savings, has two opposite effects on s_0^E. The substitution effect calls for a reduction in s_0^E, whereas the income effect triggers the "precautionary motive" that calls for an increase in s_0^E. The first result in proposition 1, stating that $s_0^D > s_0^E$, is therefore explained by the dominance of the substitution effect.

Equation (28c) indicates that the ratio between $r_0^E(\ell)$ and $r_0^E(h)$ is constant and equal to β, the degree of similitude between the two potential states of the world:

(31) $\dfrac{r_0^E(\ell)}{r_0^E(h)} = \dfrac{\pi(\ell)}{\pi(h)} \equiv \beta$

hence, the higher β, the less differentiated are $r_0^E(\ell)$ and $r_0^E(h)$. When $\beta \to 1$ the potential states of the world, and therefore $r_0^E(\ell)$ and $r_0^E(h)$, converge. This means that for $\beta \to 1$, the long-term saving contracts offered by the two alternative financial systems converge, and therefore $s_0^E \to s_0^D$. Given households' risk-aversion, a higher differentiation between $r_0^E(\ell)$ and $r_0^E(h)$ - that is, a lower β - exerts two opposite effects on s_0^E. The income effect of risk – the precautionary motive - calls for the augmentation of s_0^E. The substitution effect of risk calls for the reduction of s_0^E. Let $V(r_0^E)$ denote the variance of the state-contingent rate of return on long-term savings under EFS, which reflects the risk EFS casts onto households. Given equation (28c):

$$(32)\quad V(r_0^E) \equiv \theta[r_0^E(h)-1]^2 + (1-\theta)[r_0^E(\ell)-1]^2 = \frac{\theta(1-\theta)[\pi(h)-\pi(\ell)]^2}{[\theta\pi(h)+(1-\theta)\pi(\ell)]^2}$$

$$= \frac{\theta(1-\theta)(1-\beta)^2}{[\theta+(1-\theta)\beta]^2}$$

so that:

$$(33)\quad \frac{\partial V(r_0^E)}{\partial \beta} = -\frac{2\theta(1-\theta)(1-\beta)}{[\theta+(1-\theta)\beta]^3} < 0$$

Equation (33) states that β has a negative effect on the risk faced by households under EFS. This result is quite intuitive: the higher β, the more differentiated the potential rates of return on long-term savings, and hence, the greater the risk households face. The finding that s_0^E is increasing in β indicates the dominance of the substitution effect of risk, that is, a negative effect of risk on s_0^E.

Aggregate technological risk is also affected by θ, the probability for state h. Under DFS long-term saving contracts are fixed and therefore invariant to θ, whereas under EFS long-

term saving contracts are state-contingent so that s_0^E is affected by θ. Equation (28c) indicates that both $r_0^E(\ell)$ and $r_0^E(h)$ decrease in θ:

(34) $$\frac{dr_0^E(\ell)}{d\theta} = -\frac{\pi(\ell)[\pi(h)-\pi(\ell)]}{\bar{\pi}^2} = -\frac{\beta(1-\beta)}{[\theta+(1-\theta)\beta]^2} < 0$$

$$\frac{dr_0^E(h)}{d\theta} = -\frac{\pi(h)[\pi(h)-\pi(\ell)]}{\bar{\pi}^2} = -\frac{1-\beta}{[\theta+(1-\theta)\beta]^2} < 0$$

The income effect of an increase in θ on s_0^E therefore pulls at the direction of reducing s_0^E. The substitution effect of an increase in θ on s_0^E works via θ's impact on the risk faced by households. Differentiating equation (32) with respect to θ yields:

(35) $$\frac{\partial V(r_0^E)}{\partial\theta} = \frac{\beta(1-\theta)-\theta}{[\theta+(1-\theta)\beta]^3}$$

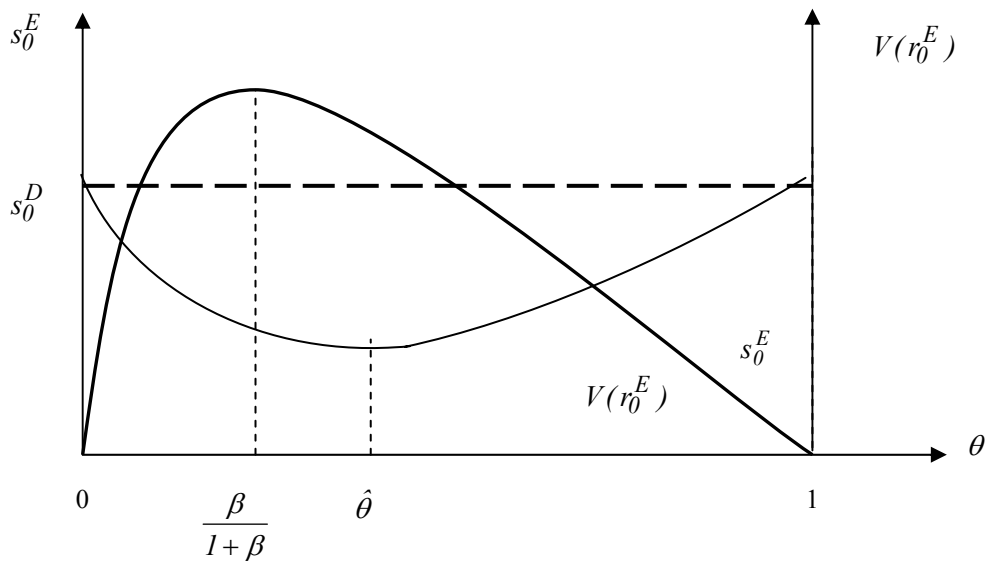
so that

$$\theta > \frac{\beta}{1+\beta} \quad iff \quad \frac{\partial V(r_0^E)}{\partial\theta} > 0 \quad , \quad \theta \le \frac{\beta}{1+\beta} \quad iff \quad \frac{\partial V(r_0^E)}{\partial\theta} \le 0$$

An increase in θ therefore increases the risk faced by households under EFS for $\theta \le \beta/(1+\beta)$, and decreases that risk for $\theta > \beta/(1+\beta)$. As risk was found to exert a negative effect on s_0^E, an increase in θ on s_0^E will negatively affect s_0^E for $\theta \le \beta/(1+\beta)$ and positively affect it for $\theta > \beta/(1+\beta)$. It follows that for $\theta \le \beta/(1+\beta)$ both the income and substitution effects generated by a rise in θ lower s_0^E; for $\theta > \beta/(1+\beta)$, the substitution effect induced by a rise in θ becomes positive, and thus contradicts the negative income effect. The findings show that s_0^E is negatively affected by θ as long as $\beta/(1+\beta) < \theta < \hat{\theta}$, thus indicating the dominance the income effect. For $\theta > \hat{\theta}$ the findings point that s_0^E is positively affected by θ, thus indicating the dominance of θ's

substitution effect. When either $\theta \to 0$ or $\theta \to 1$, EFS provides households with an almost certain return that converges to 1. It follows that for either $\theta \to 0$ or $\theta \to 1$ the long-term saving contracts offered to households under EFS converge with those offered under DFS. Consequently, $s_0^E \xrightarrow[\theta \to 0]{} s_0^D$ and $s_0^E \xrightarrow[\theta \to 1]{} s_0^D$. The effect of θ on s_0^E is illustrated by figure 3:

Figure 3: The effect of θ on s_0^E



Proposition 2.

i. *Short-term savings under EFS are counter-cyclical, that is,* $s_1^E(h) < s_1^E(\ell)$

ii. *Short-term savings under DFS are higher than short-term savings under EFS in state h, yet lower than short-term savings under EFS in state ℓ, that is,* $s_1^E(h) < s_1^D < s_1^E(\ell)$.

Proof. see the Appendix.

The intuition for the first result in proposition 2 goes as follows: the level of $s_1^E(\tilde{z})$ is affected by $r_1^E(\tilde{z})$, the state-dependent rate of return on short-term savings, and by the state-contingent value of long-term savings, $s_0^E r_0^E(\tilde{z})$. Since equation (28f) indicates that $r_1^E(\tilde{z})$ = 1 for $\tilde{z} \in \{h, \ell\}$, the level of $s_1^E(\tilde{z})$ depends exclusively upon the value of long-term savings. Equations (28d) and (28e) reveal that $r_0^E(\ell) < r_0^E(h)$, indicating that the value of long-term savings is higher in state h than in state ℓ. The value of long-term savings exerts both a (positive) income effect and a (negative) substitution effect on $s_1^E(\tilde{z})$. The findings show that $s_1^E(h) < s_1^E(\ell)$, thus indicating the dominance of the substitution effect. The behavior of short-term savings under EFS is therefore counter-cyclical: short-terms savings are high when the percentage of financially distressed projects is high, and low when the percentage of financially distressed projects is low[6]. Under DFS, short-term savings are state-invariant, and therefore neither enhance nor mitigate exogenous cyclical forces. Since households' savings are the only source of capital in the economy, it would be plausible to expect EFS to generate milder output cycles than DFS.

As for the second result in proposition 2, the intuition goes as follows. As equations (28f) and (29d) indicate that under both financial systems the rate of return on short-term savings is one, the only source of difference between $s_1^E(\tilde{z})$ and s_1^D would be the value of long-term savings. Since $r_0^E(\ell) < 1 = r_0^D$ and $s_0^E < s_0^D$ (proposition 1), it follows that the value of long-term savings under DFS is higher than that under EFS when state ℓ materializes, that is,

[6] It is however important to state that in a multi-period framework, if a negative shock in period 1 is a signal to future negative shocks, the behavior of savings could go the opposite way.

$s_0^E r_0^E(\ell) < s_0^D r_0^D$. Therefore, the negative impact of long-term savings' value on the level of short-term savings is stronger under DFS, hence $s_1^D < s_1^E(\ell)$. The proof for $s_1^E(h) < s_1^D$ is not as straightforward. As that $r_0^E(h) > 1 = r_0^D$, it is not clear at first glance whether the value of long-term savings under DFS is higher or lower than that under EFS in state *h*. In the Appendix, $s_0^D r_0^D$ is proved to be lower than $s_0^E r_0^E(h)$, thus indicating that the negative impact of long-term savings' value on the level of short-term savings is weaker under DFS, hence $s_1^E(h) < s_1^D$.

4.2.2 Output Levels and Cycles

Under both financial systems, households' savings constitute the one and only source of capital in the economy. Consequently, the comparative levels and cycles of output under DFS and EFS can be explained solely on the grounds of the comparative behavior of both long-term and short-term savings.

In period 2, projects that have survived finally mature. Given that each of these projects yields an output of *R* units of capital, the economy's aggregate output is simply *R* times the number of projects that have survived to the last period. Let $Y(\tilde{z})$ denote the level of output in state $\tilde{z}$, then:

(36) $Y(\tilde{z}) \equiv \{\lambda_0 \pi(\tilde{z}) + \lambda_0 \lambda_1(\tilde{z})[1-\pi(\tilde{z})]\}R \quad , \quad \tilde{z} \in \{h, \ell\}$

Note that equation (36) applies for both financial systems. The expression $\lambda_0 \pi(\tilde{z})$ in the right hand of equation (36) is the total number of good projects in state $\tilde{z}$, whereas $\lambda_0 \lambda_1(\tilde{z})[1-\pi(\tilde{z})]$ signifies the total number of bad salvaged projects in state $\tilde{z}$. Substituting equations (28b), (28g) and (28h) in equation (36) yields the equilibrium state-dependent level of output under EFS:

(37) $Y^E(\tilde{z}) = \left[\frac{\pi(\tilde{z})s_0^E}{1-\varepsilon} + \frac{s_1^E(\tilde{z})}{\alpha}\right]R \quad , \quad \tilde{z} \in \{h, \ell\}$

Substituting equations (10), (12), (29b), (29i) and (29j) in equation (36) yields the equilibrium state-dependent of output under DFS:

(38) $Y^D(\tilde{z}) = \left[\frac{\pi(\tilde{z})s_0^D}{1-\varepsilon} + \frac{s_1^D}{\alpha}\right]R \quad , \quad \tilde{z} \in \{h, \ell\}$

In this model, the expected level of output could be interpreted as potential output. Let $\bar{Y}^E$ and $\bar{Y}^D$ denote the expected output under EFS and DFS respectively; then, by substituting (28b) in (37) and (29b) in (38) and taking expectations one obtains:

(39) $\bar{Y}^E = Rs_0^E\left[\frac{\bar{\pi}}{1-\varepsilon} - \frac{1}{2\alpha}\right] + \frac{\gamma_1 R}{2\alpha}$

(40) $\bar{Y}^D = Rs_0^D\left[\frac{\bar{\pi}}{1-\varepsilon} - \frac{1}{2\alpha}\right] + \frac{\gamma_1 R}{2\alpha}$

Proposition 3.

i. *Under both financial systems state h output (henceforth output peak) is higher than state ℓ output (henceforth output trough)*

ii. *Output peak is higher under DFS than under EFS, that is, $Y^E(h) < Y^D(h)$*

iii. *Output trough is higher under EFS than under DFS if*

$$s_0^E > s_0^D\left[\frac{2\alpha\pi(\ell) - 1 + \varepsilon}{2\alpha\bar{\pi} - 1 + \varepsilon}\right]\frac{\bar{\pi}}{\pi(\ell)}$$

iv. *Expected (potential) output is higher under DFS than under EFS.*

Proof. See the Appendix.

Equations (37) – (38) present the state-dependent output levels under the two alternative financial systems as being determined by long-term and short-term savings. Long-term savings constitute the capital supply to the corporate sector in the initial period, and thus, determine the number of projects that eventually start up. Out of the $s_0/(1-\varepsilon)$ projects that are established in period 0, a fraction $\pi(\tilde{z})$ are of good quality, and thus will generate $\left[\pi(\tilde{z})Rs_0\right]/(1-\varepsilon)$ units of capital in the final period. The positive effect of long-term savings' on output under both financial systems is therefore strengthened the higher entrepreneurs' capital endowment ε, and the larger the state-contingent fraction of good projects in the economy, $\pi(\tilde{z})$. Short-term savings too positively affect output under both financial systems. As short-term savings constitute the capital supply to financially distressed projects, the number of salvaged projects is $s_1^E(\tilde{z})/\alpha$ under EFS and s_1^D/α under DFS; accordingly, short-term savings' contribution to output will be $s_1^E(\tilde{z})R/\alpha$ under EFS and $s_1^D R/\alpha$ under DFS. Note that in the case of EFS, short-term savings' effect on output is counter-cyclical (see proposition 2), whereas under DFS this effect is state-invariant. Under both financial systems, a high α reduces short-term savings' positive effect on output.

Let ΔY denote the peak-trough gap, then:

(41) $\Delta Y \equiv Y(h) - Y(\ell)$

Substituting (37) into (41) for both states of the world yields the equilibrium expression for the gap between the levels of output in state h and ℓ under EFS:

$$(42)\ \Delta Y^E = \left\{\frac{\left[\pi(h)-\pi(\ell)\right]s_0^E}{1-\varepsilon} - \frac{s_1^E(\ell)-s_1^E(h)}{\alpha}\right\}R$$

Substituting (38) into (41) for both states of the world gives the equilibrium expression for the gap between the levels of output in state h and ℓ under DFS:

(43) $\Delta Y^D = \dfrac{[\pi(h)-\pi(\ell)]Rs_0^D}{1-\varepsilon}$

Proposition 4. *The peak-trough gap generated by DFS is larger than that generated by EFS.*

Proof. Given that $s_0^E < s_0^D$ (proposition 1) and that $s_1^E(h) < s_1^E(\ell)$ (proposition 2), equations (42) and (43) indicate that $\Delta Y^D > \Delta Y^E$. ■

The intuition for this result lies in the comparative behavior of savings under the two alternative financial systems. Given that $s_0^E < s_0^D$ (proposition 1), the number of projects that start-up in the first period under DFS is larger than under EFS - $s_0^D/(1-\varepsilon)$ versus $s_0^E/(1-\varepsilon)$. Therefore, the reduction in the number of good quality projects when shifting from state h to state ℓ is larger under DFS than under EFS: $[\pi(h)-\pi(\ell)]s_0^D/(1-\varepsilon)$ versus $[\pi(h)-\pi(\ell)]s_0^E/(1-\varepsilon)$; hence, the gap in the output of good-quality projects is larger under DFS than under EFS. Given that $s_1^E(h) < s_1^E(\ell)$, the shift from state h to state ℓ raises the number of salvaged bad-quality projects from $s_1^E(h)/\alpha$ to $s_1^E(\ell)/\alpha$, thus creating a counter-cyclical effect. Under DFS, short-term savings are state-invariant; therefore, the number of salvaged bad quality projects is s_1^D/α regardless of the state of the world. It follows that under DFS short-term savings neither reinforce nor mitigate the exogenous cyclical forces. It follows that the comparative behavior of short-term savings further reduces the peak-trough gap under EFS with respect to that under DFS.

4.2.4 Welfare Implications

Proposition 5. *Households' lifetime utility under DFS is higher than their expected lifetime utility under EFS.*

Proof. The two financial systems differ only with respect to the long-term saving contracts they offer to households: state-contingent contracts under EFS, versus fixed contracts under DFS, where the expected return on long-term savings under EFS is equal to the fixed return under DFS (see equation (30)). Given households' risk aversion, their expected utility will be higher under DFS than under EFS.■

In the last period entrepreneurs consume their corporate gains. Let $G^E(\tilde{z})$ stand for entrepreneurs' equilibrium corporate gains under EFS in state $\tilde{z} \in \{h, \ell\}$, then:

(44) $G^E(\tilde{z}) \equiv \lambda_0^E \pi(\tilde{z})(1-\phi_0^E)R + \lambda_0^E \lambda_1^E(\tilde{z})[1-\pi(\tilde{z})](1-\phi_1^E)R$

Substituting equations (28e) – (28h) in equation (44) and rearranging yields:

(45) $G^E(\tilde{z}) = \pi(\tilde{z}) s_0^E \left[\frac{R}{1-\varepsilon} - \frac{R+\alpha}{2\alpha\bar{\pi}}\right] + \frac{\gamma_1(R-\alpha)}{2\alpha}$

Let $\overline{G}^E$ stand for entrepreneurs' expected gains under EFS, then:

(46) $\overline{G}^E = s_0^E \left[\frac{\bar{\pi}R}{1-\varepsilon} - \frac{R+\alpha}{2\alpha}\right] + \frac{\gamma_1(R-\alpha)}{2\alpha}$

Let $G^D(\tilde{z})$ denote entrepreneurs' equilibrium corporate gains under DFS in state $\tilde{z} \in \{h, \ell\}$, then:

(47) $G^D(\tilde{z}) \equiv \lambda_0^D \pi(\tilde{z})[R-(1-\varepsilon)\rho_0^D] + \lambda_0^D \lambda_1^D(\tilde{z})[1-\pi(\tilde{z})][R-(1-\varepsilon)\rho_0^D - \alpha\rho_1^D(\tilde{z})]$

Substituting equations (29e) – (29h) in equation (47) and rearranging yields:

(48) $G^D(\tilde{z}) = s_0^D \left[\frac{\pi(\tilde{z})R}{1-\varepsilon} - \frac{R+\alpha}{2\alpha}\right] + \frac{\gamma_1(R-\alpha)}{2\alpha}$

Let $\overline{G}^D$ stand for entrepreneurs' expected gains under DFS, then:

(49) $\overline{G}^{D} = s_0^D \left[\frac{\overline{\pi} R}{1-\varepsilon} - \frac{R+\alpha}{2\alpha} \right] + \frac{\gamma_1 (R-\alpha)}{2\alpha}$

Proposition 6. *Entrepreneurs' welfare is higher under DFS than under EFS.*

Proof. See Appendix.

The intuition for this result goes as follows: EFS induces the risk-averse agents – households - to provide insurance to the risk-neutral agents - entrepreneurs. As the households require an appropriate compensation for sharing corporate sector's risks, entrepreneurs would rather not be insured at all – which is the case under DFS.

Proposition 7. *Total welfare in the economy is higher under DFS than under EFS.*

Proof. Combine propositions 5 and 6. ■

Proposition 7 states that DFS Pareto-dominates EFS. The intuition behind this result is simple. EFS creates an inefficient situation in which the risk-averse agents – households – provide insurance to the risk-neutral agents – entrepreneurs. It is important to note that although the welfare implications of the model are straightforwardly implied by the configuration of agents' risk-aversion (the assumption that households are risk-averse and entrepreneurs are risk-neutral), they will also hold under the more general (and realistic) assumption that entrepreneurs' risk-aversion is milder than households'. It is also important to point that the relaxation of the assumption that households and entrepreneurs are separate entities (that is, that households do not own projects) may lead to different welfare statements.

5. Concluding Remarks

The paper shows that the financial system's configuration affects real activity levels and cycles. This non-neutrality stems from the different patterns of risk sharing induced by each financial system and the resulting behavior of savings, set by consumption-smoothing risk-averse households. By offering households state-contingent long-term saving contracts, EFS induces them to share the corporate sector's aggregate technological risk. DFS, by contrast, provides households with fixed long-term saving contracts, thus isolating them from technological uncertainties. Given that households are risk-averse, each financial system yields a different consumption-smoothing saving's path. DFS produces higher long-term savings than EFS; hence the number of projects that are established under DFS is larger. In the interim, EFS generates counter-cyclical short-term savings, whereas DFS produces state-invariant short-term savings that neither enhance nor mitigate the exogenous cyclical forces; consequently, the peak-trough gap is milder under EFS than under DFS. As large output cycles are usually conceived as undesired phenomena, this result raises an important point in favor of EFS. But the results also raise a number of arguments against EFS. EFS generates lower output peaks than DFS and under some conditions, deeper output troughs. Potential output is lower under EFS. Welfare wise, EFS creates an inefficient risk-sharing pattern that induces the risk-averse sector (households) to share the risks of the risk-neutral sector (entrepreneurs). Consequently, EFS is Pareto-dominated by DFS, which insulates the two sectors. This result holds as long as households are assumed to be more risk-averse than entrepreneurs – which seems to be the case in the real world. In the end of the day, the paper does not imply that one financial system should be preferred to the other; this question should be dealt with within a more comprehensive discussion that takes into account not only real activity and welfare considerations but also financial stability, political aspects etc. The contribution of the paper to the equity-versus-debt debate is in focusing on the so far untreated facet of real activity and welfare implications of financial systems, and in presenting the pros and cons of each system on the ground of a theoretical model. It is

however important to point that the results could be sensitive to the specific configurations of relative bargaining power, agents' set of alternatives etc. A challenge for future research would be to overcome the limitations of the present model whilst preserving the ability to draw analytical results.

Appendix

i. Solving the Equity Version of the Model

In equilibrium both (7) and (8) hold with strict equality. Substituting (8) for both states of the world in (7) yields:

(i.1) $\phi_0^E = \dfrac{1-\varepsilon}{\bar{\pi}R}$

where $\bar{\pi} \equiv \theta\pi(h)+(1-\theta)\pi(\ell)$. It can be seen that the equilibrium share offered to the fund by a typical firm in period 0 is decreasing in ε, the amount of capital invested by the entrepreneur in his own firm, and R, the gross return generated by a project that survives to period 2. ϕ_0^E is also a decreasing function of $\pi(h)$, $\pi(\ell)$ and θ, given that $\pi(h) > \pi(\ell)$. Substituting (i.1) in (8) gives:

(i.2) $\phi_1^E(\tilde{z}) = \phi_1^E = \dfrac{\alpha}{R} \quad , \quad \tilde{z} \in \{h, \ell\}$

It can be seen from equation (i.2) that in equilibrium $\phi_1^E(\tilde{z})$ is not state-dependent and can therefore be denoted by ϕ_1^E; in period 1, a financially distressed firm issues the same share for both states of the world. Since $R > \alpha$, it is obvious that ϕ_1^E is smaller than 1.

Substituting (8), (10), (17) and (i.1) in (15) gives:

(i.3) $r_0^E(\tilde{z}) = \dfrac{\pi(\tilde{z})}{\bar{\pi}} + \dfrac{B_0^E}{s_0^E}\left[1 - \dfrac{\pi(\tilde{z})}{\bar{\pi}}\right] \quad , \quad \tilde{z} \in \{h, \ell\}$

Since $\pi(\ell) < \pi(h)$ it follows from equation (i.3) that $r_0^E(\ell) < r_0^E(h)$, that is, the gross return rate on households' period 0 savings is higher if the state of the world is revealed to be h. Equation (i.5) also indicates that $r_0^E(h)$ is increasing in $\pi(h)$ and decreasing in $\pi(\ell)$, and accordingly $r_0^E(\ell)$ is increasing in $\pi(\ell)$ and decreasing in $\pi(h)$.

Substituting (14) and (i.3) in (5), taking account of restriction (3e) and rearranging yields:

$$\text{(i.4)}\quad s_1^E(\tilde{z}) = max\{\frac{\gamma_1}{2} - \frac{\pi(\tilde{z})s_0^E}{2\bar{\pi}} - \frac{B_0^E}{2}\left[1 - \frac{\pi(\tilde{z})}{\bar{\pi}}\right],\ 0\},\qquad \tilde{z} \in \{h, \ell\}$$

Since $\pi(\ell) < \pi(h)$ it follows from equation (i.4) that $s_1^E(\ell) \geq s_1^E(h)$, that is, the typical household saves more in period 1 if the materialized state of the world is ℓ.

Substituting (10) in (18) yields:

$$\text{(i.5)}\quad B_1^E(\tilde{z}) = \max\{s_1^E(\tilde{z}) - \frac{\alpha(s_0^E - B_0^E)[1-\pi(\tilde{z})]}{1-\varepsilon},\ 0\}\qquad ,\qquad \tilde{z} \in \{h, \ell\}$$

Substituting (i.4) and (1.5) in (17) and rearranging yields:

$$\text{(i.6)}\quad \lambda_1^E(\tilde{z}) = \begin{cases} min\{\dfrac{max\{\bar{\pi}\gamma_1 - \pi(\tilde{z})s_0^E - B_0^E[\bar{\pi} - \pi(\tilde{z})], 0\}(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(\tilde{z})](s_0^E - B_0^E)},\ 1\} & if\ \ s_1^E(\tilde{z}) > 0 \\ 0 \quad if\ \ s_1^E(\tilde{z}) = 0 \end{cases}$$

$$\tilde{z} \in \{h, \ell\}$$

Substituting (i.3) and (i.4) for both states of the world in (4) yields an equation in s_0^E. However, in case $B_0^E > 0$, the equation thus yielded is a high-order equation in s_0^E; in case $B_0^E = 0$ the equation thus yielded is a quadratic equation in s_0^E:

$$\text{(i.7)}\quad 3\pi(\ell)\pi(h)s_0^{E2} + \{\gamma_1\bar{\pi}[\pi(\ell) + \pi(h)] - 2\gamma_0\pi(\ell)\pi(h) + 2\gamma_1\bar{\pi}^2\}s_0^E + \gamma_1\bar{\pi}^2(\gamma_1 - 2\gamma_0) = 0$$

Equation (i.7) can be simplified to

$$\text{(i.8)}\quad 3\beta s_0^{E2} + \{\gamma_1(1+\beta)[\theta + (1-\theta)\beta] - 2\gamma_0\beta + 2\gamma_1[\theta + (1-\theta)\beta]^2\}s_0^E$$
$$+ \gamma_1(\gamma_1 - 2\gamma_0)[\theta + (1-\theta)\beta]^2 = 0$$

Where $\beta = \frac{\pi(\ell)}{\pi(h)}$. The solution of (i.8) gives s_0^E in terms of θ, β, γ_0 and γ_1. There are two potential solutions for s_0^E, and therefore two potential equilibria for the equity version of the model.

ii. Solving the Debt Version of the Model

In equilibrium both (22) and (23) hold with strict equality. Substituting (23) in (22) yields:

(ii.1) $\theta\pi(h)\rho_0(h)+(1-\theta)\pi(\ell)\rho_0(\ell)=1$

Substituting (ii.1) in (23) for state h gives:

(ii.2) $\rho_1^D(h)=1-\left[\frac{1-(1-\theta)\pi(\ell)\rho_0^D(\ell)}{\alpha\theta\pi(h)}\right](1-\varepsilon)$

Rearranging (36) for state ℓ yields:

(ii.3) $\rho_1^D(\ell)=1-\frac{\rho_0^D(\ell)(1-\varepsilon)}{\alpha}$

Substituting (10), (11) (17) and (23) in (27) yields:

(ii.4) $\rho_0^D(\tilde{z})=\frac{r_0 s_0^D - B_0^D}{\pi(\tilde{z})[s_0^D - B_0^D]}$

Equation (ii.4) indicates that

(ii.5) $\frac{\rho_0^D(h)}{\rho_0^D(\ell)}=\frac{\pi(\ell)}{\pi(h)}$

Substituting (ii.5) in (ii.1) yields:

(ii.6) $\rho_0^D(\ell)=\frac{1}{\pi(\ell)}$

Substituting (ii.6) in (ii.1) yields:

(ii.7) $\rho_0^D(h) = \dfrac{1}{\pi(h)}$

Substituting (ii.6) in (23):

(ii.8) $\rho_1^D(\ell) = 1 - \dfrac{1-\varepsilon}{\alpha\pi(\ell)}$

Substituting (ii.7) in (23):

(ii.9) $\rho_1^D(h) = 1 - \dfrac{1-\varepsilon}{\alpha\pi(h)}$

Substituting (10), (11) (17), (23) and (ii.7) in (27) for state *h* (arbitrary) yields:

(ii.10) $r_0^D = 1$

Substituting (26) and (ii.10) in (21), taking account of restriction (19e):

(ii.11) $s_1^D(\tilde{z}) = s_1^D = max\ \{ \dfrac{\gamma_1 - s_0^D}{2}\ ,\ 0 \}$

Substituting (26), (ii.10) and (ii.11) in (20) taking account of restriction (19d) yields:

(ii.12) $s_0^D = min\{ \dfrac{2\gamma_0 - \gamma_1}{3}, \dfrac{\gamma_0}{2} \}$

Substituting (ii.12) in (ii.11):

(ii.13) $s_1^D = max\ \{ \dfrac{2\gamma_1 - \gamma_0}{3}\ ,\ 0 \}$

It can be easily seen that if $\gamma_0 < 2\gamma_1$ then $s_0^D = \dfrac{2\gamma_0 - \gamma_1}{3}$ and $s_1^D = \dfrac{2\gamma_1 - \gamma_0}{3}$, whereas if

$\gamma_0 \geq 2\gamma_1$, then $s_0^D = \dfrac{\gamma_0}{2}$ and $s_1^D = 0$.

Substituting (ii.12) in (10) gives:

(ii.14) $\lambda_0^D = min\{ \dfrac{2\gamma_0 - \gamma_1}{3(1-\varepsilon)}\ ,\ \dfrac{\gamma_0}{2(1-\varepsilon)}\ ,\ 1 \}$

Substituting (ii.13) and (ii.14) in (19):

$$\text{(ii.15)}\quad \lambda_1^D(\tilde{z}) = \begin{cases} \min\{\dfrac{(2\gamma_1-\gamma_0)(1-\varepsilon)}{\alpha(2\gamma_0-\gamma_1)[1-\pi(\tilde{z})]}\ ,\ 1\ \}\ \text{if}\ \ s_1^D = \dfrac{2\gamma_1-\gamma_0}{3} \\ 0 \quad \text{if} \quad s_1^D = 0 \end{cases}$$

$$\tilde{z} \in \{h, \ell\}$$

III. Proofs of Lemmas & Propositions

Proof of Lemma 1: Equation (28a) can be expressed as $a(s_0^E)^2 + bs_0^E + c = 0$, where $a \equiv 3\pi(h)\pi(\ell)$, $b \equiv \gamma_1\bar{\pi}[\pi(h)+\pi(\ell)+2\bar{\pi}] - 2\gamma_0\pi(h)\pi(\ell)$ and $c \equiv \gamma_1\bar{\pi}^2(\gamma_1 - 2\gamma_0)$. It can be seen that *a* and *c* are positive[7] whereas *b* could be either positive or negative. The two potential solutions for s_0^E are given by:

$$(s_0^E)_1 = \frac{-b+\left|\sqrt{b^2-4ac}\right|}{2a} \quad ; \quad (s_0^E)_2 = \frac{-b-\left|\sqrt{b^2-4ac}\right|}{2a}$$

Since $-4ac > 0$, it follows that $b < \left|\sqrt{b^2-4ac}\right|$ so that $(s_0^E)_1 > 0$ and $(s_0^E)_2 < 0$. The equilibrium solution for s_0^E is therefore $(s_0^E)_1$.

Proof of lemma 2: A prerequisite for the existence of equilibrium in general for the equity version of the model is that projects' shares held by the fund are within range and that the fund's share in bad salvaged projects is larger than its share in good quality projects. This is summarized by the requirement that $0 < \phi_0 < \phi_1 < 1$. Given that $\varepsilon < 1$ equation (28e)

[7] A prerequisite for the existence of an internal equilibrium is $\gamma_1 > 2\gamma_0$.

indicates that ϕ_0 is strictly positive. Given that $\alpha < R$, it is always true that $\phi_1 < 1$. Equations (28e) and (28f) indicate that $\phi_0 < \phi_1$ if and only if $\varepsilon > 1 - \alpha\bar{\pi}$.

In addition to the above prerequisites for the existence of equilibrium, an internal equilibrium for the model under EFS requires that $s_0^E > 0$, $s_1^E(\tilde{z}) > 0$, $0 < \lambda_0^E < 1$ and $0 < \lambda_1^E(\tilde{z}) < 1$, $\tilde{z} \in \{h, \ell\}$. As the analytical solution of s_0^E is unattainable, so is the condition on the parameters that yield $s_0^E > 0$. Equation (28b) indicates that $s_1^E(\tilde{z}) > 0$ if $s_0^E < \dfrac{\bar{\pi}\gamma_1}{\pi(\tilde{z})}$. Since this condition has to hold for both states of the world, and as $\pi(\ell) < \pi(h)$, a sufficient condition is that $s_0^E < \dfrac{\bar{\pi}\gamma_1}{\pi(h)}$. Equation (28g) indicates that since $\varepsilon < 1$, λ_0^E is positive if s_0^E is positive; the condition for $\lambda_0^E < 1$ is that $s_0^E < 1 - \varepsilon$. Equation (28h) indicates that $\lambda_1^E(\tilde{z})$ is positive if $s_1^E(\tilde{z})$ is positive, whereas the condition for $\lambda_1^E(\tilde{z}) < 1$ is that $s_0^E < \dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)] + \pi(h)(1-\varepsilon)}$. Consequently, the prerequisites for the existence of an internal equilibrium under EFS are summarized by $\dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)] + \pi(h)(1-\varepsilon)} < s_0^E < min\{1-\varepsilon, \dfrac{\bar{\pi}\gamma_1}{\pi(h)}\}$. This prerequisite is plausible only if $\dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)] + \pi(h)(1-\varepsilon)} < min\{1-\varepsilon, \dfrac{\bar{\pi}\gamma_1}{\pi(h)}\}$. As in equilibrium $\varepsilon > 1 - \alpha\bar{\pi}$, it follows that $\dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)] + \pi(h)(1-\varepsilon)} < \dfrac{\bar{\pi}\gamma_1}{\pi(h)}$. However, in case $1-\varepsilon < \dfrac{\bar{\pi}\gamma_1}{\pi(h)}$, the condition for $\dfrac{\bar{\pi}\gamma_1(1-\varepsilon)}{2\alpha\bar{\pi}[1-\pi(h)] + \pi(h)(1-\varepsilon)} < 1-\varepsilon$ is that $\gamma_1 < \alpha[2 - \pi(h)]$.

An internal equilibrium of the debt version of the model is characterized by $s_0^D > 0$, $s_1^D > 0$, $0 < \lambda_0^D < 1$ and $0 < \lambda_1^D(\tilde{z}) < 1$, for $\tilde{z} \in \{h, \ell\}$. Equation (29a) indicates that $s_0^D > 0$ if and only if $\gamma_1 < 2\gamma_0$. Equation (29b) indicates that the condition for $s_1^D > 0$ is that $\gamma_1 > \frac{\gamma_0}{2}$. According to equation (29g), λ_0^D is positive if s_0^D is positive; whereas the condition for $\lambda_0^D < 1$ is that $\gamma_1 > 2\gamma_0 - 3(1-\varepsilon)$. Equation (29h) indicates that $\lambda_1^D(\tilde{z})$ is positive if s_1^D is positive, whereas the condition for $\lambda_1^D(\tilde{z}) < 1$ is $\gamma_1 < \frac{\gamma_0\{2\alpha[1-\pi(\tilde{z})]+1-\varepsilon\}}{\alpha[1-\pi(\tilde{z})]+2(1-\varepsilon)}$. It can be seen that since $\frac{\gamma_0\{2\alpha[1-\pi(\tilde{z})]+1-\varepsilon\}}{\alpha[1-\pi(\tilde{z})]+2(1-\varepsilon)} < 2\gamma_0$, the condition for existence of internal equilibrium for the debt version of the model is:

$$\max\{\frac{\gamma_0}{2}, 2\gamma_0 - 3(1-\varepsilon)\} < \gamma_1 < \frac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}$$

This condition is plausible if $\max\{\frac{\gamma_0}{2}, 2\gamma_0 - 3(1-\varepsilon)\} < \frac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}$. It can be easily seen that $\frac{\gamma_0}{2} < \frac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}$, *whereas the condition for* $2\gamma_0 - 3(1-\varepsilon) < \frac{\gamma_0\{2\alpha[1-\pi(h)]+1-\varepsilon\}}{\alpha[1-\pi(h)]+2(1-\varepsilon)}$ is that $\varepsilon < 1 - \frac{\gamma_0 - \alpha[1-\pi(h)]}{2}$. Therefore, in case $2\gamma_0 - 3(1-\varepsilon) > \frac{\gamma_0}{2}$, an additional requirement is that $\varepsilon < 1 - \frac{\gamma_0 - \alpha[1-\pi(h)}{2}$.

Proof of Proposition 1:

(i) The right hand of equation (28a) is a polynum $P(\cdot)$ in s_0^E. Following the proof for lemma 1, it can be seen that $\frac{\partial P(\cdot)}{\partial s_0^E} = 2as_0^E + b$, so that $\left.\frac{\partial P(\cdot)}{\partial s_0^E}\right|_{(s_0^E)_1} = \left|\sqrt{b^2 - 4ac}\right|$ > 0 and $\left.\frac{\partial P(\cdot)}{\partial s_0^E}\right|_{(s_0^E)_2} = -\left|\sqrt{b^2 - 4ac}\right|$ < 0, and that $\frac{\partial^2 P(\cdot)}{\partial s_0^{E\,2}} = 2a$ > 0. Consequently, $P(\cdot)$ is U-shaped. Therefore, in order to prove that in equilibrium $s_0^E < s_0^D$ one has to show that $P(\cdot)$ has a positive value at s_0^D. Substituting s_0^D in $P(\cdot)$ yields $\frac{\gamma_1\theta(1-\theta)(2\gamma_0 - \gamma_1)[\pi(h) - \pi(\ell)]^2}{3\pi(h)^2}$ > 0; therefore $s_0^E < s_0^D$.

(ii) Substituting $\beta \equiv \frac{\pi(\ell)}{\pi(h)}$ into $P(\cdot)$ yields a polynum $P'(\cdot)$ of s_0^E, with coefficients *a', b'* and *c'* that are practically identical to *a, b* and *c,* respectively. The derivative of s_0^E with respect to β is $\frac{\partial s_0^E}{\partial \beta} = -\frac{\partial P'(\cdot)}{\partial \beta} \Big/ \frac{\partial P'(\cdot)}{\partial s_0^E}$. Since $\frac{\partial P'(\cdot)}{\partial s_0^E}$ > 0 at the vicinity of s_0^E, the next step in proving that s_0^E is an increasing function of β is to show that $\frac{\partial P'(\cdot)}{\partial \beta}$ < 0. Calculating $\frac{\partial P'(\cdot)}{\partial \beta}$ and rearranging yields:

$$\frac{\partial P'(\cdot)}{\partial \beta} = (3s_0^E + \gamma_1 - 2\gamma_0)[2\gamma_1(1-\theta)\beta + s_0^E] + 2\gamma_1\theta(1-\theta)(1-\beta)(2s_0^E + \gamma_1 - 2\gamma_0)$$

Using the result $s_0^E < s_0^D$ and recalling that $s_0^D = \frac{2\gamma_0 - \gamma_1}{3}$ one can see that $3s_0^E < 2\gamma_0 - \gamma_1$ and hence, that $2s_0^E < 2\gamma_0 - \gamma_1$. Consequently, $\frac{\partial P'(\cdot)}{\partial \beta} < 0$ so that $\frac{\partial s_0^E}{\partial \beta} >$ 0. Substituting $\beta = 0$ in (28a) yields $s_0^E = \frac{\theta(2\gamma_0 - \gamma_1)}{1+2\theta} = \frac{3\theta}{1+2\theta} s_0^D$, whereas substituting β = 1 yields $s_0^E = \frac{2\gamma_0 - \gamma_1}{3} = s_0^D$. Given that s_0^E is monotonically increasing in β, $s_0^E \xrightarrow[\beta \to 0]{} \frac{3\theta}{1+2\theta} s_0^D$ and $s_0^E \xrightarrow[\beta \to 1]{} s_0^D$.

(iii) Equation (31a) indicates that s_0^D is invariant to θ. Substituting either θ =0 or θ =1 in (28a) yields $s_0^E = \frac{2\gamma_0 - \gamma_1}{3} = s_0^D$. The derivative of s_0^E with respect to θ is $\frac{\partial s_0^E}{\partial \theta} = -\frac{\partial P'(\cdot)}{\partial \theta} \Big/ \frac{\partial P'(\cdot)}{\partial s_0^E}$. It has already been shown that $\frac{\partial P'(\cdot)}{\partial s_0^E} > 0$. Differentiating $P'(\cdot)$ with respect to θ yields $\frac{\partial P'(\cdot)}{\partial \theta} = \gamma_1(1+\beta)\{2(2s_0^E + \gamma_1 - 2\gamma_0)[\theta + (1-\theta)\beta] + (1+\beta)s_0^E\}$. Therefore, $\frac{\partial P'(\cdot)}{\partial \theta} > 0$ iff $s_0^E > \frac{2(2\gamma_0 - \gamma_1)[\theta + (1-\theta)\beta]}{1+\beta+4[\theta+1-\theta)\beta]} = \frac{6[\theta+(1-\theta)\beta]}{1+\beta+4[\theta+(1-\theta)\beta]} s_0^D$ and vice versa for $\frac{\partial P'(\cdot)}{\partial \theta} < 0$. Using the result $s_0^E < s_0^D$, it can be seen that a sufficient though not necessary condition for $\frac{\partial P'(\cdot)}{\partial \theta} < 0$, and hence for $\frac{\partial s_0^E}{\partial \theta} > 0$, is that $\theta > 0.5$. Given that $s_0^E = s_0^D$ for either θ =0 or θ =1 and that $\frac{\partial P'(\cdot)}{\partial \theta}$ is continuous in θ, there is some non-negative threshold value of θ that is smaller than 0.5, $\hat{\theta}$:

$$\hat{\theta} = \frac{1}{1-\beta}\left\{\frac{(1+\beta)s_0^E(\hat{\theta})}{2[3s_0^D - 2s_0^E(\hat{\theta})]} - \beta\right\}$$

such that:

$$\frac{\partial s_0^E}{\partial \theta} < 0 \quad for \quad 0 \le \theta < \hat{\theta}$$

$$\frac{\partial s_0^E}{\partial \theta} = 0 \quad for \quad \theta = \hat{\theta}$$

$$\frac{\partial s_0^E}{\partial \theta} > 0 \quad for \quad \hat{\theta} < \theta \le 1$$

The value of $\hat{\theta}$ depends on β. Since $s_0^E \xrightarrow[\beta \to 0]{} \frac{3\theta}{1+2\theta} s_0^D$ one can easily see that $\hat{\theta} \xrightarrow[\beta \to 0]{} 0$. In the same manner, given that $s_0^E \xrightarrow[\beta \to 1]{} s_0^D$ one obtains $\hat{\theta} \xrightarrow[\beta \to 1]{} 0.5$. However, the derivative of $\hat{\theta}$ with respect to β is rather complicated, and does not enable a deduction about its sign:

$$\frac{\partial \hat{\theta}}{\partial \beta} = \frac{1}{1-\beta}\left\{\frac{\left[s_0^E + (1+\beta)\frac{\partial s_0^E}{\partial \beta}\right]}{2\left(3s_0^D - 2s_0^E\right)} + \frac{(1-2\beta)(1+\beta)s_0^E}{2(1-\beta)\left(3s_0^D - 2s_0^E\right)^2} - \frac{1}{1-\beta}\right\}.$$

Proof of proposition 2: Given that $\pi(\ell) < \pi(h)$, equation (28b) indicates that $s_1^E(h) < s_1^E(\ell)$.

Substituting equations (29c) and (29d) in equation (21) yields $s_1^D = \frac{\gamma_1 - s_0^E}{2}$; given that $s_0^E < s_0^D$ (proposition 1) and $\pi(\ell) < \pi(h)$, equation (28b) indicates that:

$$s_1^D < s_1^E(\ell)$$

$$s_1^D > s_1^E(h) \quad iff \quad s_0^E > \frac{\overline{\pi}}{\pi(h)} s_0^D \Leftrightarrow s_0^E > [\theta + (1-\theta)\beta] s_0^D$$

In order to prove that $s_0^E > [\theta + (1-\theta)\beta] s_0^D$, one has to show that the positive root of $P(\cdot)$ (see proof of proposition 1) is larger than $[\theta + (1-\theta)\beta] s_0^D$, or, alternatively, that $P(\cdot)$ is negative at the point $[\theta + (1-\theta)\beta] s_0^D$. Substituting $s_0 = [\theta + (1-\theta)\beta] s_0^D$ in $P(\cdot)$ yields $2\gamma_0\beta(1-\theta)(\beta-1) + 2\gamma_1(1-\theta)(\beta-1)[\theta+(1-\theta)\beta]$, which given that $\beta < 1$ is negative. Therefore, $s_1^D > s_1^E(h)$.

Proof of Proposition 3:

Equation (37) indicates that $Y^E(h) > Y^E(\ell)$ *iff* $\dfrac{\pi(h) - \pi(\ell)}{1-\varepsilon} s_0^E > \dfrac{s_1^E(\ell) - s_1^E(h)}{\alpha}$.

Substituting equation (29b) in the above condition and rearranging yields:

$$Y^E(h) > Y^E(\ell) \;\; iff \;\; \varepsilon > 1 - 2\alpha\overline{\pi}$$

Since one of the requirements for the existence of an internal equilibrium for both the EFS and DFS models is $\varepsilon > 1 - \alpha\overline{\pi}$ (see lemma 2), it is obvious that $Y^E(h) > Y^E(\ell)$.

Given that $\pi(\ell) < \pi(h)$, equation (38) indicates that $Y^D(h) > Y^D(\ell)$.

Equations (37) and (38) indicate that:

$$Y^E(h) < Y^D(h) \;\; iff \;\; \frac{s_1^E(h) - s_1^D}{\alpha} < \frac{\pi(h)(s_0^D - s_0^E)}{1-\varepsilon}$$

Given that $s_1^E(h) < s_1^D$ (proposition 2) and that $s_0^E < s_0^D$ (proposition 1), the condition is obviously fulfilled.

Equations (37) and (38) indicate that:

$$Y^E(\ell) > Y^D(\ell) \;\; iff \;\; \frac{\pi(\ell) s_0^E}{1-\varepsilon} + \frac{s_1^E(\ell)}{\alpha} > \frac{\pi(\ell) s_0^D}{1-\varepsilon} + \frac{s_1^D}{\alpha}$$

Substituting equations (21), (28b), (29c) and (29d) in the above condition and rearranging yields:

$$Y^E(\ell) > Y^D(\ell) \;\; iff \;\; s_0^E > \left[\frac{2\alpha\pi(\ell) - 1 + \varepsilon}{2\alpha\bar{\pi} - 1 + \varepsilon}\right]\frac{\bar{\pi}}{\pi(\ell)} s_0^D$$

Finally, given that $s_0^E < s_0^D$ (proposition 1) equations (39) and (40) indicate that expected (potential) output is higher under DFS than under EFS.

Proof of Proposition 6: As entrepreneurs are risk-neutral, their expected gains can be used as a welfare measure. Given that $s_0^E < s_0^D$ (proposition 1) equations (46) and (49) indicate that entrepreneurs' expected gains are higher under DFS. Therefore, entrepreneurs' welfare is higher under DFS.

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